Functional Analysis

Anne Thomas
Honours Course, Semester 1 2017

March 2017

1 Course description

Modern functional analysis is the study of infinite-dimensional vector spaces and linear transformations between such spaces. Thus it can be thought of as linear algebra in an infinite dimensional setting.

The main topics we will cover are:

1. Normed vector spaces, completions and Banach spaces
2. Linear operators and the operator norm
3. Hilbert spaces and the Stone–Weierstrass Theorem
4. Uniform boundedness and the Open Mapping Theorem
5. Dual spaces and the Hahn–Banach Theorem
6. Spectral theory

If time permits, we may discuss an extra topic related to my research at the end, namely Kazhdan's Property (T).

Each of the above 6 topics has an associated Exercise Sheet, available on the course webpage. We will not have formal tutorials but you should work through the exercises on these sheets.

The course webpage has Daniel Daners' lecture notes "Introduction to Functional Analysis" as an extra resource. We will be covering some topics in a different order to these notes or with a different emphasis.

2 Lectures

We will have two one-hour lectures each week, usually Monday 12:00–13:00 and Tuesday 10:00–11:00, in Carslaw 830. There are some weeks where I won't be available on the Monday and we'll find another time that everyone can attend.

3 Prerequisites

The formal prerequisites are MATH3961 Metric Spaces (Advanced) and MATH3969 Measure Theory and Fourier Analysis (Advanced). You will also need to recall some basic linear algebra in general vector spaces, from MATH2961 Linear Algebra and Vector Calculus (Advanced).

4 Assessment

Three assignments each worth 10%, due at the end of weeks 4, 8 and 12, and one closed-book exam worth 70%. Assignment questions will be selected from the Exercise Sheets, and the exam may also contain questions from the Exercise Sheets.

As usual, you are encouraged to work on exercises with other students, but in a small enough group that everyone can make a contribution, and you should write up your assignment solutions independently.

References

[1] J. Conway, *A Course in Functional Analysis*, Springer, 1990.

[2] K. Yosida, *Functional Analysis*, Springer, 1980.

[3] R.J. Zimmer, *Essential Results of Functional Analysis*, University of Chicago Press, 1990.