1. Let \( \theta: \mathbb{R}^3 \rightarrow \mathbb{R}^2 \) be defined by \( \theta \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \). Calculate the matrix of \( \theta \) relative to the bases \( d = \left( \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \\ 2 \\ -2 \\ 4 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \end{pmatrix} \) of \( \mathbb{R}^3 \) and \( \mathbb{R}^2 \).

2. (i) Let \( \phi: \mathbb{R}^2 \rightarrow \mathbb{R} \) be defined by \( \phi \begin{pmatrix} x \\ y \end{pmatrix} = (1 \ 2) \begin{pmatrix} x \\ y \end{pmatrix} \). Calculate the matrix of \( \phi \) relative to the bases \( c = \left( \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \end{pmatrix} \right) \) of \( \mathbb{R}^2 \) and \( \mathbb{R} \).

   (ii) With \( \phi \) as in (i) and \( \theta \) as in Exercise 1 calculate \( \phi \theta \) and its matrix relative to the two given bases. Hence verify that \( M_{bd}(\phi \theta) = M_{bc}(\phi) M_{cd}(\theta) \).

3. Suppose that \( \theta: \mathbb{R}^6 \rightarrow \mathbb{R}^4 \) is a linear transformation with kernel of dimension 2. Is \( \theta \) surjective?

4. For each of the following linear transformations calculate the dimensions of the kernel and image, and check that your answers are in agreement with the Main Theorem on Linear Transformations.

   (i) \( \theta: \mathbb{R}^4 \rightarrow \mathbb{R}^2 \) given by \( \theta \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 2 & -1 & 3 & 5 \\ 1 & -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \).

   (ii) \( \theta: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) given by \( \theta \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ -2 & 1 \\ -6 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \).

   (iii) \( \theta: V \rightarrow V \) given by \( \theta(p(x)) = p'(x) \), where \( V \) is the space of all polynomials over \( \mathbb{R} \) of degree less than or equal to 3.

5. Is it possible to find a \( 3 \times 2 \) matrix \( A \), a \( 2 \times 2 \) matrix \( B \) and a \( 2 \times 3 \) matrix \( C \) such that

\[
ABC = \begin{pmatrix} 2 & 2 & 2 \\ 3 & 3 & 0 \\ 4 & 0 & 0 \end{pmatrix}.
\]

6. Let \( V \) and \( W \) be finitely generated vector spaces of the same dimension and let \( \theta: V \rightarrow W \) be a linear transformation. Use the Main Theorem on Linear Transformations to prove that \( \theta \) is injective if and only if it is surjective.