

## Comments on Tutorial 2 Exercises and Solutions

1. This generalises easily to rings. Let  $R$  is a ring and  $S$  a set with an addition and multiplication. Then if  $\varphi : R \rightarrow S$  respects multiplication and addition then the image of  $\varphi$  is ring with addition and multiplication compatible with those of  $S$ . Along the way you So if in fact the addition and multiplication in  $S$  make  $S$  a ring,  $\varphi$  is a ring homomorphism and the image of  $\varphi$  is a subring of  $S$  This is proved in the notes in section 5 see in particular Theorem 5.8 and Proposition 5.11.

In lectures I took the more common less sophisticated approach of only dealing with ring homomorphism. The approach of the notes has a pay-off later in simplifying establishing the ring structure of quotient rings.

2. The example given is in fact a ring and maybe the question should have been give an example of a ring  $S$  with a left inverse but not a right inverse. If we do not require  $S$  to be a ring, a simple example is take any set  $S$  with more than one element and define  $ab = a$  for all  $a, b \in S$ . Then every element is a left identity but there is no right identity.
3. You may or may not notice that in the solution there is reference to the sum of 3 rings. Given our definitions (see Notes section 6) the solutions should say product of 3 rings. What is going on?

In fact there is notation of a sum of rings. The direct sum  $\bigoplus_{i \in I} R_i$  of a family of rings indexed by some set  $I$  is the subring of  $\prod_{i \in I} R_i$  of elements  $(r_i)_{i \in I}$  with  $r_i = 0$  for all but finitely many  $i$ . When  $I$  is finite the direct product and direct sum are the same.

- 6.(b) Since this a subset of  $\mathbb{Q}$  all that is required is to show  $S$  is subring. The solution proves closure under addition and multiplication. The solution neglects to establish the subset is closed under taking negatives or to point out that this set is non-empty.
7. Equivalently observe

$$(1 - x)(1 + x + \cdots + x^{n-1}) = 1 - x^n = 1 - 1 = 0.$$

So  $1 - x$  and  $1 + x + \cdots + x^{n-1}$  are zero divisors.