

Comments on Tutorial 3 Exercises and Solutions

1. $-(-a) = a$ is the additive version of $(x^{-1})^{-1} = x$, valid in any group.
5. You can characterise $I + J$ as the smallest ideal of R containing the ideal I and J , and $I \cap J$ as the largest ideal of R contained in both of the ideals I and J .

Note that demonstration that IJ is closed under addition need not be so elaborate. Just note that IJ is simply all finite sums in R of terms of the form ab with $a \in I$ and $b \in J$ and that the sum of two such sums is patently another such sum.

6. (Implicit challenge)

Convince yourself that in general for $a, b \in \mathbb{Z}$, $a, b > 0$, $a\mathbb{Z} + b\mathbb{Z} = d\mathbb{Z}$ where $d = (a, b)$ is the g.c.d of a and b , $IJ = ab\mathbb{Z}$ and $a\mathbb{Z} \cap b\mathbb{Z} = m\mathbb{Z}$, where m is lowest common multiple of a and b .

8. More generally show for R a ring with identity that $I \leftrightarrow M_n(I)$ is a 1-1 correspondence between the ideals of R and the ideals of $M_n(R)$. In particular if F is a field a matrix ring $M_n(F)$ $n \geq 1$ has exactly two ideals, the zero ideal and the whole matrix ring. Note question 7(ii) is the case $n = 1$.