THE UNIVERSITY OF SYDNEY

MATH3962 Rings, Fields and Galois Theory (Advanced)

Semester 1 **Exam Information Sheet** 2009

The exam will have five questions. Each will be of equal value. The questions will each be marked out of 20. Part marks will be indicated.

Part of the first question will ask for some examples:

Q1(a) *For each of the following either give an example (no justification required), or quote a definition or result to show no such example exists.*

Otherwise answering in the questions you should justify your answers and state results from lectures and tutorials as you use them.

MATH3962 Key Topics

General Ring Theory:

Rings, commutative, identity elements, zero divisors, characteristic.
Matrix rings, polynomial rings.
Homomorphisms, kernel, image.
Ideals and quotient rings. First isomorphism theorem.
Maximal ideals, prime ideals.

Integral Domains:

Fields of Fraction. Units, associates, divisibility, irreducibles, primes, greatest common divisors, principal ideal domains, unique factorisation domains.
Polynomials over fields, and over the integers,
Main applications: Irreducibilty of polynomials over the rationals.
The Gaussian integers, and representing primes as a sum of two squares.

Fields:

Field extensions, algebraic elements, minimal polynomials, simple algebraic extensions as quotient rings.
Degree of an extension.
Application: Constructible real numbers, and constructibilty by ruler and compass.

Galois Theory:

Groups of automorphisms, fixed fields, Galois extensions. Radical extensions, simple radical extensions, simple radical Galois extensions.
Application: Non solvability of the general degree 5 equation by radicals.

Group Theory:

Finite cyclic groups and their sub-groups and quotient groups. Symmetric groups. Solvable and non-solvable groups.