THE UNIVERSITY OF SYDNEY

MATH3962: Rings, Fields and Galois Theory

Semester 1	**Homework 1**	2009

This homework exercise is set for you get feedback on writing out proofs of elementary results.

If you would like some help , or would to discuss your ideas, come and see my in Carslaw 527 at one of my timetabled consultation hours, Tuesday 12:00-1:00 or Wednesday 3:30-4:30, or at any other mutually convenient time.

1. Let R be a ring with an identity element 1.
Let $U(R)$ denote the set of elements of R which have a two sided multiplicative inverse.
Show that $U(R)$ is a group under ring multiplication.

Recall a group G has to (1) a multiplication operation, (2) the multiplication has to be associative, (3) G has to have a two sided identity and (4) every element in G must have a multiplicative inverse.

There were four things to establish.

(1) $U(R)$ is closed under multiplication of elements in R, i.e. if x and y have a two-sided inverse, so does xy.
(2) Multiplication is associative in $U(R)$.
(3) (a) $1 = 1_R \in U(R)$ *and then*
(b) that it is an identity for $U(R)$.
(4) That if $u \in U(R)$ i.e u has a two-sided inverse for multiplication in R, that
(a) this inverse is in $U(R)$ *and then*
(b) that it is an inverse for u for multiplication in $U(R)$.