

THE UNIVERSITY OF SYDNEY  
MATH3962: RINGS, FIELDS AND GALOIS THEORY

Semester 1

**Homework 3**

2009

This homework exercise is set for you get feedback on writing out proofs of elementary results.

If you would like some help getting started, or would to discuss your ideas, come and see my in Carslaw 527 at one of my timetabled consultation hours, Tuesday 12:00-1:00 or Wednesday 3:30-4:30, or at any other mutually convenient time.

*Due: Thursday 23rd April at the beginning of the 2:00 pm Lecture*

1. Let  $R$  be a commutative ring with identity element 1 and  $a, b \in R$ .
  - (i) Show  $a|b \Leftrightarrow bR \subseteq aR$ .  
Note only one of the implication uses  $R$  has an identity element.
  - (ii) Deduce  $a|b$  and  $b|a$  if and only if  $aR = bR$ .
  
2. Let  $R$  be an integral domain and  $a, b \in R$ .
  - (i) Show that for  $a, b \in R$ ,  $a|b$  and  $b|a$  if and only if  $a$  and  $b$  are associates.
  - (ii) Give an example of a commutative ring  $R$  with an identity element together with two elements  $a, b \in R$  with  $a|b$  and  $b|a$  which are not associates.