THE UNIVERSITY OF SYDNEY
MATH3962: Rings, Fields and Galois Theory

Semester 1	**Homework 4**	2009

This homework exercise is set to give you timely experience of working with a Galois field.

If you would like some help getting started, or would to discuss your ideas, come and see my in Carslaw 527 at one of my timetabled consultation hours, Tuesday 12:00-1:00 or Wednesday 3:30-4:30, or at any other mutually convenient time.

Due: Thursday 30th April at the beginning of the 2:00 pm Lecture

1. (*i*) Show that $x^4 + x^3 + 1$ is irreducible in $\mathbb{F}_2[x]$.

(*ii*) List the distinct elements of the field $E = \mathbb{F}_2[\alpha]$, where $\alpha^4 + \alpha^3 + 1 = 0$.

(*iii*) Show that the powers of α run through the non-zero elements of E.

(*iv*) Hence pair each non-zero element of E with its inverse.

(*v*) Show $x^4 + x^3 + 1$ factors as a product of linear factors in $E[x]$.