

THE UNIVERSITY OF SYDNEY
MATH3962: RINGS, FIELDS AND GALOIS THEORY

Semester 1

Homework 5

2009

If you would like some help getting started, or would to discuss your ideas, come and see me in Carslaw 527 at one of my timetabled consultation hours, Tuesday 12:00-1:00 or Wednesday 3:30-4:30, or at any other mutually convenient time.

Due: Monday 1st June at the beginning of the 2:00 pm Lecture

1. Let $\phi : K \rightarrow L$ be a field embedding and F a subfield of K .

Show that if $\alpha_1, \alpha_2, \dots, \alpha_r$ generate K as an F -space, then $\phi(\alpha_1), \phi(\alpha_2), \dots, \phi(\alpha_r)$ generate $\phi(K)$ as a $\phi(F)$ -space.

(i) Show that if $\alpha_1, \alpha_2, \dots, \alpha_r \in K$ are F -linearly independent then $\phi(\alpha_1), \phi(\alpha_2), \dots, \phi(\alpha_r) \in \phi(K)$ are $\phi(F)$ -linearly independent.

(ii) Deduce $[K : F] = [\phi(K) : \phi(F)]$.

Hint: Consider the cases $[K : F]$ finite and $[K : F]$ infinite separately.