THE UNIVERSITY OF SYDNEY

SCHOOL OF MATHEMATICS & STATISTICS

MATH2063/2963 Mathematical Computing & Nonlinear Systems 2012

An example that is made simpler with computer algebra

1. (Drazin, Q3.6, p111) Consider the cubic mapping

$$x_{n+1} = ax_n - x_n^3 \quad -\infty < a, x_n < \infty\,.$$

(a) Find all fixed points, giving any conditions on a that must be satisfied for their existence.

(b) Investigate for which values of a these fixed points are linearly stable.

(c) Show that if X_1 and X_2 constitute a two-cycle, then they both satisfy the ninth degree polynomial

$$X^9 - 3aX^7 + 3a^2X^5 - (a^3 + a)X^3 + (a^2 - 1)X = 0\,.$$

Verify that this can be factorised as

$$X(X^2 - a + 1)(X^2 - a - 1)(X^4 - aX^2 + 1) = 0\,,$$

and hence find any non-trivial 2-cycles. State necessary conditions for their existence, and calculate over what range of a they are stable.

(d) Sketch the bifurcation diagram you have established so far, as in the last question. You should have found a bifurcation at $a = 1$; this type of behaviour is called a *pitchfork bifurcation*, and the example here is referred to as *supercritical.* There are also flip bifurcations.