from the consideration of the special case in which ϕ and ψ are constants κ and l respectively.

In this case the equations are

$$\left.\begin{aligned}\frac{dx}{dt} &= -\kappa xy\\ \frac{dy}{dt} &= \kappa xy - ly\\ \frac{dz}{dt} &= ly\end{aligned}\right\} \tag{29}$$

and as before $x + y + z = \mathrm{N}$.

Thus

$$\frac{dz}{dt} = l(\mathrm{N} - x - z),$$

and $\dfrac{dx}{dz} = -\dfrac{\kappa}{l}x$, whence $\log\dfrac{x_0}{x} = \dfrac{\kappa}{l}z$, since we assume that z_0 is zero.

Thus

$$\frac{dz}{dt} = l\left(\mathrm{N} - x_0 e^{-\frac{\kappa}{l}z} - z\right).$$

Since it is impossible from this equation to obtain z as an explicit function of t, we may expand the exponential term in powers of $\dfrac{\kappa}{l}z$, and we shall assume that $\dfrac{\kappa}{l}z$ is small compared with unity.

Thus

$$\frac{dz}{dt} = l\left\{\mathrm{N} - x_0 + \left(\frac{\kappa}{l}x_0 - 1\right)z - \frac{x_0\kappa^2 z^2}{2l^2}\right\}.$$

But $\mathrm{N} - x_0 = y_0$, where y_0 is small. It is for this reason that we have to take into consideration the third term in z^2, as although $\dfrac{\kappa}{l}z$ is small compared with unity, its square may not be small as compared with $\left(\dfrac{\kappa}{l}x_0 - 1\right)z$.

The solution of this equation is

$$z = \frac{l^2}{\kappa^2 x_0}\left\{\frac{\kappa}{l}x_0 - 1 + \sqrt{-q}\tanh\left(\frac{\sqrt{-q}}{2}lt - \phi\right)\right\} \tag{30}$$

where

$$\phi = \tanh^{-1}\frac{\frac{\kappa}{l}x_0 - 1}{\sqrt{-q}},$$

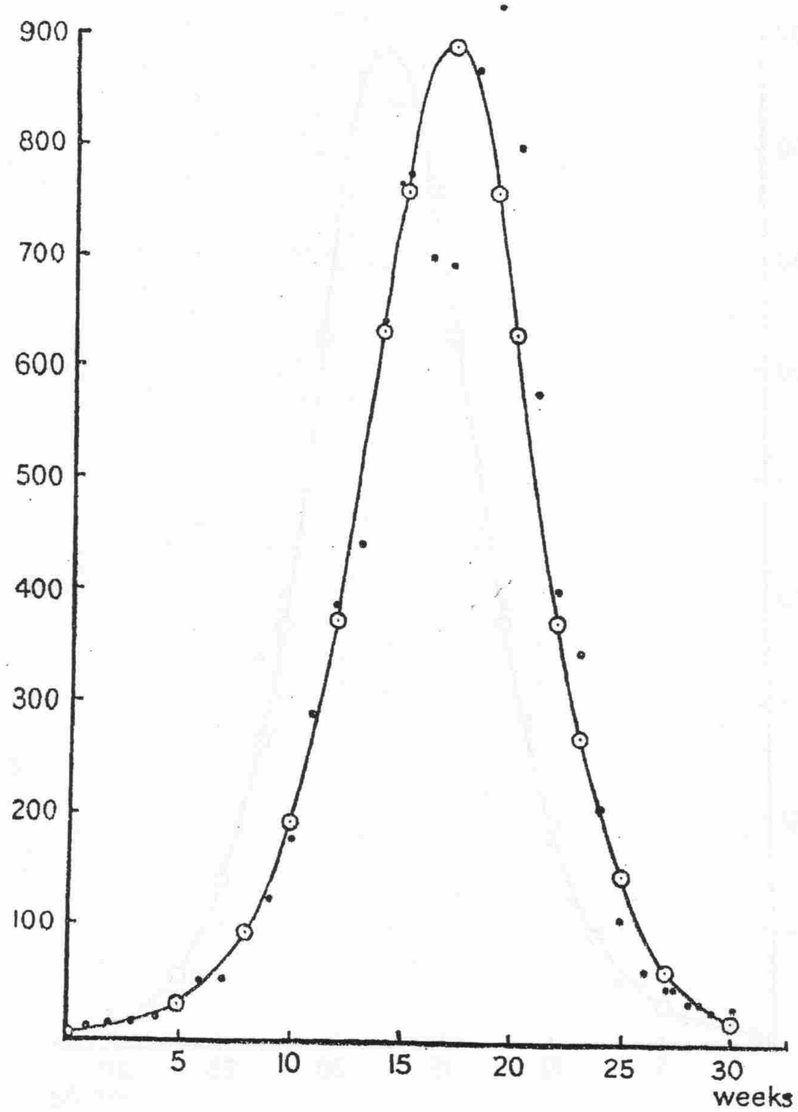
and

$$\sqrt{-q} = \left\{\left(\frac{\kappa}{l}x_0 - 1\right)^2 + 2x_0 y_0\frac{\kappa^2}{l^2}\right\}^{\frac{1}{2}}.$$

Should be $(-q)$

Also for the rate at which cases are removed by death or recovery which is the form in which many statistics are given

$$\frac{dz}{dt} = \frac{l^3}{2x_0\kappa^2}\sqrt{-q}\,\text{sech}^2\left(\frac{\sqrt{-q}}{2}lt - \phi\right). \tag{31}$$



The accompanying chart is based upon figures of deaths from plague in the island of Bombay over the period December 17, 1905, to July 21, 1906. The ordinate represents the number of deaths per week, and the abscissa denotes the time in weeks. As at least 80 to 90 per cent. of the cases reported terminate fatally, the ordinate may be taken as approximately representing dz/dt as a function of t. The calculated curve is drawn from the formula

$$\frac{dz}{dt} = 890\,\text{sech}^2\,(0{\cdot}2t - 3{\cdot}4).$$