For the various parts of questions 1 and 2, fill in just one box, corresponding to the statement which is always true.

1. *Sample multiple choice*

(a) A mapping $x_{n+1} = f(x_n, a)$, where a is a parameter, has a period-doubling bifurcation at a value $a = a^*$.

- ☐ The derivative of f with respect to a is -1 at $a = a^*$
- ☐ The derivative of f with respect to x is -1 at $a = a^*$
- ☐ For $a < a^*$ there is one fixed point, for $a > a^*$ there are two
- ☐ The derivative of f with respect to a is +1 at $a = a^*$

(b) The mapping $x_{n+1} = x_n^2/4 + x_n/2$ has

- ☐ no unstable fixed points
- ☐ unstable fixed points at $x = 0$ and $x = -2$
- ☐ one stable fixed point at $x = 0$, one unstable at $x = 2$
- ☐ a stable fixed point at $x = 2$

(c) The logistic map $x_{n+1} = f(x_n) = ax_n(1 - x_n)$, where $0 \leq a \leq 4$ is a real constant and $0 \leq x_n \leq 1$, has a period quadrupling bifurcation when

- ☐ $a = 3$
- ☐ $a = 1$
- ☐ $a = 1 + \sqrt{6}$
- ☐ $a = 3.5699456....$

(d) A certain self-similar fractal set requires 2^n elements of length $1/4^n$ to cover it at any stage n of its construction. The box counting dimension is

- ☐ $1/2$
- ☐ $\log 2/(2 \log n)$
- ☐ $\log 2/ \log 3$
- ☐ $2/3$.

2. *Sample multiple choice*

(a) A two-dimensional map has a fixed point where the Jacobian matrix is evaluated to be

$$\begin{pmatrix} 4 & 3 \\ -2 & -1 \end{pmatrix}$$

This fixed point

- ☐ is a stable node
- ☐ has indefinite stability
- ☐ is a stable focus
- ☐ is unstable

(b) In the complex z plane, the mapping $z_{n+1} = z_n^2 + c$ has a Julia set

- ☐ only for values of c in the Mandelbrot set
- ☐ which is fractal for $c = 0$
- ☐ which contains an unstable fixed point if $c = 0$
- ☐ which contains a stable fixed point if $c = 0.1$

(c) An autonomous 2–dimensional linear system of differential equations is defined by the real coefficient matrix A. The condition on the eigenvalues of A which ensures the system has an *unstable focus* at the origin is:

- ☐ one eigenvalue is complex and positive
- ☐ one eigenvalue has modulus > 1, the other has modulus < 1
- ☐ both eigenvalues are real and positive
- ☐ the eigenvalues are complex with real part greater than zero.

(d) For the Lorenz equations $dx/dt = \sigma(y - x),\ dy/dt = rx - xz - y,\ dz/dt = xy - bz$, the three fixed points when $r = 2, \sigma = 10, b = 8/3$

- ☐ are all stable
- ☐ are on the point of undergoing a Hopf bifurcation
- ☐ have one negative and two positive Lyapunov exponents
- ☐ are two stable, one unstable.