

Assignment 1

4/PG: Partial Differential Equations

Semester 2, 2007

Lecturer: Daniel Daners

Due on **Monday, September 17** by **4pm** in Carlaw **Room 715**

(Slide under the door when locked).

Late assignments are not accepted without *prior arrangement* well before the deadline!

You must attach the signed cover-sheet to the front of your assignment (see over)!

1. Let B be the unit ball in \mathbb{R}^N , $N \geq 2$. Show that

$$u(x) := \log(\log(1 + |x|^{-1}))$$

is an element of $W_N^1(B)$. In particular argue that $H^1(B) \not\subset L_\infty(B)$.

2. Let $F \in C^1(\mathbb{R})$ with $\|F'\|_\infty < \infty$. Suppose that $1 < p < \infty$ and that Ω is bounded. Show that $F \circ u \in W_p^1(\Omega)$ for all $u \in W_p^1(\Omega)$, and that the chain rule

$$\frac{\partial}{\partial x_i}(F \circ u) = (F' \circ u) \frac{\partial u}{\partial x_i}$$

applies to the weak derivatives.

Hint: Use approximation by smooth functions.

3. (a) Apply the above question to show that $|u| \in H^1(\Omega)$ for all $u \in H^1(\Omega)$. Also compute the weak partial derivatives of $|u|$. Show the same applies in $H_0^1(\Omega)$.

Hint: For $\varepsilon > 0$ let $F_\varepsilon(\xi) := \sqrt{x^2 + \varepsilon^2} - \varepsilon$, use the above question and let $\varepsilon \rightarrow 0$.

- (b) Show that the positive and negative parts of $u \in H^1(\Omega)$ are also in $H^1(\Omega)$.

4. (a) Suppose that $u \in H^1(\Omega) \cap C(\bar{\Omega})$. If $u = 0$ on $\partial\Omega$, show that $u \in H_0^1(\Omega)$

Hint: Fix a function $F \in C^1(\mathbb{R})$ with $|F(\xi)| \leq |\xi|$ for all $\xi \in \mathbb{R}$, $F(\xi) = 0$ for $|\xi| \leq 1$ and $F(\xi) = \xi$ for $|\xi| \geq 2$. Define $u_n := \frac{1}{n}F(nu)$, show that $u_n \in H_0^1(\Omega)$ and that $u_n \rightarrow u$ in $H^1(\Omega)$

- (b) If $u \in C^2(\Omega) \cap C^1(\bar{\Omega})$ satisfies

$$-\Delta u = f \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial\Omega,$$

show that u is a weak solution of the above equation.

Assignment Cover Sheet

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Family Name

Given Names **SID**

Some collaboration between students on assignments is encouraged, since it can be a real aid to understanding. Thus it is legitimate for students to discuss assignment questions at a general level, provided everybody involved makes some contribution. However, students should produce their own individual written solution. Copying someone else's work is plagiarism, and is unacceptable. The University may impose severe penalties in cases where plagiarism is detected.

I certify that:

- I have read and understood the *University of Sydney Student Plagiarism: Coursework Policy and Procedure* at <http://www.maths.usyd.edu.au/u/UG/Plagiarism.pdf>.
- this assignment is all my own work, and that no part of this assignment has been copied from another person.
- I have not allowed my work to be copied by another person.

Signature **Date**

This part to be completed by the marker:

Question 1 out of 5

Question 2 out of 5

Question 3 out of 8

Question 4 out of 7

Grand total out of 25