

Assignment 1

4/PG: Partial Differential Equations

Semester 2, 2007

Lecturer: Daniel Daners

Due on **Friday, 2 November** by **4pm** in Carlaw **Room 715**

(Slide under the door when locked).

Late assignments are not accepted without *prior arrangement* well before the deadline!

You must attach the signed cover-sheet to the front of your assignment (see over)!

1. Consider the boundary value problem

$$\begin{aligned} -\Delta u + \lambda u &= f \quad \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} + \beta u &= 0 \quad \text{on } \partial\Omega, \end{aligned} \tag{1}$$

where $\beta \geq 0$ is constant and ν the outward pointing unit normal to the bounded connected set Ω of class C^1 .

- (a) Define what is meant by a weak solution of (1), including the relevant Hilbert space and bilinear form associated with the problem.
- (b) Prove that (1) has a unique weak solution for all $\lambda > 0$ and $f \in L_2(\Omega)$.

2. Suppose that $u \in C^2(\Omega) \cap C(\bar{\Omega})$ satisfies the differential inequality

$$\begin{aligned} -\Delta u &\geq 0 \quad \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} + \beta u &\geq 0 \quad \text{on } \partial\Omega, \end{aligned} \tag{2}$$

where $\beta \geq 0$ is constant and ν the outward pointing unit normal to the bounded connected set Ω of class C^2 . Show that either $u \equiv 0$ or $u(x) > 0$ for all $x \in \bar{\Omega}$. In case $\beta > 0$ show that u is constant.

Assignment Cover Sheet

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Family Name

Given Names **SID**

Some collaboration between students on assignments is encouraged, since it can be a real aid to understanding. Thus it is legitimate for students to discuss assignment questions at a general level, provided everybody involved makes some contribution. However, students should produce their own individual written solution. Copying someone else's work is plagiarism, and is unacceptable. The University may impose severe penalties in cases where plagiarism is detected.

I certify that:

- I have read and understood the *University of Sydney Student Plagiarism: Coursework Policy and Procedure* at <http://www.maths.usyd.edu.au/u/UG/Plagiarism.pdf>.
- this assignment is all my own work, and that no part of this assignment has been copied from another person.
- I have not allowed my work to be copied by another person.

Signature **Date**

This part to be completed by the marker:

Question 1 out of 8

Question 2 out of 6

Grand total out of 12