Preliminary Reading:
Chapter 2 of the Linear Algebra book.

Objectives:
By the end of Week 7, to achieve at least a pass level, you should be able to
(7a) compute sums and products of matrices
(7b) compute with row and column vectors
(7c) reduce matrices to row echelon form and to reduced row echelon form

To achieve higher than a pass level you should be able to
(7d) use the “sigma notation” for summation of series
(7e) prove the basic laws of matrix algebra
(7f) find all matrices satisfying given conditions

Preparatory questions. (Answers are on the next page.)

1. Given the matrices $C = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $D = \begin{bmatrix} 4 & 5 \\ 6 & 1 \end{bmatrix}$, find $C + D$, $3C$, $CD$ and $DC$.

2. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -2 & 2 & -6 \\ 7 & 12 & 17 \end{bmatrix}$. Use elementary row operations to transform $A$ to a reduced row echelon matrix.

3. Write the column vector $\begin{bmatrix} 2 + s + 3t \\ 1 - t \\ s \\ -5 \end{bmatrix}$ as a sum of the form $a + sb + tc$, where $a$, $b$ and $c$ are column vectors.

Self-assessment checklist
Tick the box or boxes and seek help from your tutor, if required.

☐ I was unable to complete the Preparatory Questions.

☐ I completed the Preparatory Questions:
  ☐ with ease.  ☐ with some effort.  ☐ with difficulty.
Practice questions

4. Let $A = \begin{bmatrix} -2 & 1 & 1 \\ -2 & 0 & 1 \\ -4 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 0 & 2 \\ 4 & 1 & -2 \\ 6 & 0 & -2 \end{bmatrix}$. Compute $A^3$, $BA$ and $AB$.

5. Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ and $D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$. Suppose that $AD = DA$, and suppose also that no two of $a$, $b$, $c$ are equal. Show that $a_{12}$, $a_{21}$, $a_{13}$, $a_{31}$, $a_{23}$ and $a_{32}$ are all zero.

6. Prove that $(A + B)C = AC + BC$ whenever $A$ and $B$ are $r \times n$ matrices and $C$ is an $n \times p$ matrix.

7. Let $x$ be a $1 \times 3$ row vector and $y$ a $3 \times 1$ column vector. Given that $yx = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 2 & -6 \\ 3 & -3 & 9 \end{bmatrix}$, determine $xy$.

8. Express $v = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$. That is, find numbers $x$, $y$ and $z$ such that $v = xv_1 + yv_2 + zv_3$.

Answers to Preparatory Questions

1. $C + D = \begin{bmatrix} 6 & 4 \\ 9 & 3 \end{bmatrix}$, $3C = \begin{bmatrix} 6 & -3 \\ 9 & 6 \end{bmatrix}$, $CD = \begin{bmatrix} 2 & 9 \\ 24 & 17 \end{bmatrix}$, and $DC = \begin{bmatrix} 23 & 6 \\ 15 & -4 \end{bmatrix}$.

2. The row operations are

$$
\begin{bmatrix}
1 & 1 & 1 \\
-2 & 2 & -6 \\
7 & 12 & 17 \\
\end{bmatrix}
\xrightarrow{R_2:=R_2+2R_1}
\begin{bmatrix}
1 & 1 & 1 \\
0 & 4 & -4 \\
0 & 5 & 10 \\
\end{bmatrix}
\xrightarrow{R_2:=\frac{1}{4}R_2}
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -1 \\
0 & 5 & 10 \\
\end{bmatrix}
\xrightarrow{R_3:=R_3-5R_2}
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -1 \\
0 & 0 & 15 \\
\end{bmatrix}
\xrightarrow{R_3:=\frac{1}{15}R_3}
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -1 \\
0 & 0 & 1 \\
\end{bmatrix}
\xrightarrow{R_1:=R_1+R_3}
\begin{bmatrix}
1 & 1 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\xrightarrow{R_1:=R_1-R_2}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
$$
3.
\[
\begin{bmatrix}
2 + s + 3t \\
1 - t \\
s \\
-5
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
1 \\
-5
\end{bmatrix}
+ 
\begin{bmatrix}
1 \\
0 \\
1 \\
0
\end{bmatrix}
+ t 
\begin{bmatrix}
3 \\
-1 \\
0 \\
0
\end{bmatrix}
\]

Self-assessment checklist:

Think about the work you have completed and how it relates to the objectives on the first page. This is aimed at helping you focus on how well you are going and on the areas in which you may need to do further practice or seek assistance.

In the following table, each row corresponds to one of the objectives listed on the first page. Tick the box corresponding to the level of understanding you believe you have achieved.

<table>
<thead>
<tr>
<th>Objective (7a)</th>
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<th>little</th>
<th>good</th>
<th>very good</th>
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<td>Objective (7f)</td>
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Web Quiz

There are additional self assessment tasks on the Web. Go to the Web page at


and then do the Web Quiz for Week 7.