THE UNIVERSITY OF SYDNEY
MATH3076 AND MATH3976 (ADVANCED)

Mathematical Computing Numerical Methods 2012

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Set 5 — Initial Value Problems for ODE's

For the week beginning Monday 28th May.

* — tutorial question; A — advanced; S — supplementary

1* (a) Attached is a listing of the file `rk4.inc` containing an incomplete main program `rk4main`, which calls the subroutine `rk4`. Subroutine `rk4` implements the classical fourth-order Runge-Kutta method over one interval of width h to solve the initial value problem

$$\frac{dy}{dt} = f(t, y)\,, \quad y(t_0) = y_0\,.$$

Note the use of the `open` statement,

`open (unit=8, file=output, status='unknown')`

in the main program to open a file and connect it to unit 8. The name of the file is given by the character variable `output`, which is input from the keyboard. The main program writes t and y into unit 8 every `nprint` steps, where `nprint` is an `integer` read in with t_0, y_0, t_n and n. The `nprint` feature is implemented using the intrinsic function `mod(j,nprint)`, which gives the remainder of `j` divided by `nprint`.

Copy the file `$mc3nm/rk4.inc` to `rk4.f90` in your directory and complete the main program to do n steps from t_0 to $t_n = t_0 + nh$, where h is the step-size. Determine h from t_0,t_n and n.

(b) Test your program by solving the simple initial value problem

$$\frac{dy}{dt} = -2ty\,, \quad y(0) = 1\,,$$

($0 \le t \le 4$) and then comparing with the analytic solution. Plot your results using funplot.

(c) Modify your program to solve

$$\frac{dy}{dt} = -A(y - \sin t) + \cos t\,, \quad y(0) = 1\,,$$

on $[0, 3.5]$ with $A = 1000$. What stepsize must you take? Plot your results.

(d) Write a simple program to solve the initial value problem in Part (c) using the backward Euler (1-step Gear) method and the linearity of the differential equation,

$$y_{n+1} = y_n + hf(t_{n+1}, y_{n+1})\,.$$

Take $A = 1000$. How does the stepsize compare with Part (c). What happens if $A = 10000$?

2. Given the initial value problem

$$y' = -2ty\,, \qquad y(0) = 1\,,$$

find y when $t = 0.2$ by the following methods:

(a) analytically;

(b) Taylor series (find answer to four decimal places);

(c) Euler's method, with

  (*i*) $h = 0.2$ and

  (*ii*) $h = 0.1$;

(d) two-stage second-order Runge-Kutta method, with $h = 0.2$;

(e) classical four-stage fourth-order Runge-Kutta method, with $h = 0.2$;

(f) Euler-trapezoidal predictor-corrector method, with $h = 0.2$. The trapezoidal corrector is

$$y_{k+1} = y_k + \frac{1}{2}\{f(t_k, y_k) + f(t_{k+1}, y_{k+1})\}\,.$$

Give the results for

  (*i*) one and

  (*ii*) two applications of the corrector.

[Anwers: (a) $y = e^{-t^2}$, $y(0.2) = 0.9607894.$; (b) Six terms gives 0.96080; (c) (i) 1, (ii) 0.98; (d) 0.96; (e) 0.9607893; (f) (i) 0.96, (ii) 0.9616.]

3. *3-Step Gear's Method.* The 3-step method of Gear for stiff problems is of the form

$$y_{n+1} = a_1 y_n + a_2 y_{n-1} + a_3 y_{n-2} + hb_0 f(t_{n+1}, y_{n+1})\,.$$

(a) Give a condition that determines the coefficients a_1, a_2, a_3, b_0. Use this condition to find a_1, a_2, a_3, b_0.

(b) Is the method implicit or explicit ? Suggest a suitable technique for implementing this method on stiff problems.