(A)

MATH1111 Lecture 5 Wed 22/3/2017

Two important formulae

Squares:

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2$$

$$= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2,$$

i.e.,

$$\boxed{(a+b)^2 = a^2 + 2ab + b^2}$$

Then

$$(a-b)^2 = (a+(-b))^2$$

$$= a^2 + 2a(-b) + (-b)^2$$

$$= a^2 - 2ab + b^2$$

i.e.,

$$\boxed{(a-b)^2 = a^2 - 2ab + b^2}$$

Useful forms involving a variable x & constant c:

$$\boxed{\begin{aligned}(x+c)^2 &= x^2 + 2xc + c^2 \\ (x-c)^2 &= x^2 - 2xc + c^2\end{aligned}}$$

<u>Practice</u>: Expand the following

$$(x+1)^2 = x^2 + 2x + 1,$$

$$(x-1)^2 = x^2 - 2x + 1,$$

$$(x+2)^2 = x^2 + 4x + 4,$$

$$(x-2)^2 = x^2 - 4x + 4,$$

$$(x+\tfrac{1}{2})^2 = x^2 + x + 1/4,$$

$$(x-\tfrac{1}{2})^2 = x^2 - x + 1/4,$$

$$(x+\tfrac{3}{2})^2 = x^2 + 3x + 9/4,$$

$$(x-\tfrac{3}{2})^2 = x^2 - 3x - 9/4.$$

<u>Difference of two squares</u>:

Observe

$$(a+b)(a-b) = a^2 - ab + ba - b^2$$

$$= a^2 - \cancel{ab} + \cancel{ab} - b^2$$

$$= a^2 - b^2,$$

i.e.

$$\boxed{(a^2 - b^2) = (a+b)(a-b),}$$

Useful forms involving x:

$$\boxed{x^2 - c^2 = (x+c)(x-c)}$$

$$\boxed{x^2 - k = (x+\sqrt{k})(x-\sqrt{k})} \quad \text{if } k \geq 0,$$

<u>Practice</u>: factorize the following:

$$x^2 - 9 = x^2 - 3^2 = (x+3)(x-3)$$

$$x^2 - 4 = x^2 - 2^2 = (x+2)(x-2)$$

$$x^2 - \tfrac{1}{4} = x^2 - \left(\tfrac{1}{2}\right)^2 = \left(x+\tfrac{1}{2}\right)\left(x-\tfrac{1}{2}\right)$$

$$x^2 - 2 = (x+\sqrt{2})(x-\sqrt{2})$$

$$x^2 - \tfrac{1}{2} = \left(x+\sqrt{\tfrac{1}{2}}\right)\left(x-\sqrt{\tfrac{1}{2}}\right) = \left(x+\tfrac{1}{\sqrt{2}}\right)\left(x-\tfrac{1}{\sqrt{2}}\right).$$

<u>Important technique</u>: <u>rationalising the denominator</u>

– using difference of squares formula to remove <u>surds</u> from denominators

expressions involving $\sqrt{\ }$

Example:

$$\frac{1+\sqrt{2}}{1-\sqrt{2}} = \frac{(1+\sqrt{2})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})}$$

$$= \frac{1+2\sqrt{2}+(\sqrt{2})^2}{1^2-(\sqrt{2})^2}$$

$$= \frac{1+2\sqrt{2}+2}{1-2}$$

$$= \frac{3+2\sqrt{2}}{-1}$$

$$= -3-2\sqrt{2}.$$

Practice: simplify $\frac{1+\sqrt{5}}{3-\sqrt{5}}$.

Solution: $\frac{1+\sqrt{5}}{3-\sqrt{5}} = \frac{(1+\sqrt{5})}{(3-\sqrt{5})}\cdot\frac{(3+\sqrt{5})}{(3+\sqrt{5})} = \frac{3+\sqrt{5}+3\sqrt{5}+5}{3^2-(\sqrt{5})^2}$

$$= \frac{8+4\sqrt{5}}{9-5} = \frac{8+4\sqrt{5}}{4}$$

$$= 2+\sqrt{5}.$$

E

Quadratic equations

A quadratic (expression) has the form

$$ax^2 + bx + c$$

where x is a variable and a, b, c are constants.

A quadratic equation has the form

$$\boxed{ax^2 + bx + c = 0}$$

and its solutions are called roots of the quadratic

Two main methods of solution

1. quadratic formula (see next time)

2. Factorise the quadratic, put each linear factor $= 0$ and read off solutions.

e.g. $x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\Rightarrow x-2 = 0$ or $x-1 = 0$

$\Rightarrow x = 2$ or $x = 1$.

$$(x+a)(x+b) = x^2 + xb + ax + ab$$
$$= x^2 + bx + ax + ab$$
$$= x^2 + (a+b)x + ab.$$

Practice: Solve $x^2 + 7x + 10 = 0$

Solution: $x^2 + 7x + 10 = 0$

$\Rightarrow (x+5)(x+2) = 0$

$\Rightarrow x+5 = 0$ or $x+2 = 0$

$\Rightarrow x = -5$ or $x = -2$.

Practice: Solve $x^2 + 3x - 10 = 0$

Solution: $x^2 + 3x - 10 = 0$

$\Rightarrow (x+5)(x-2) = 0$

$\Rightarrow x+5 = 0$ or $x-2 = 0$

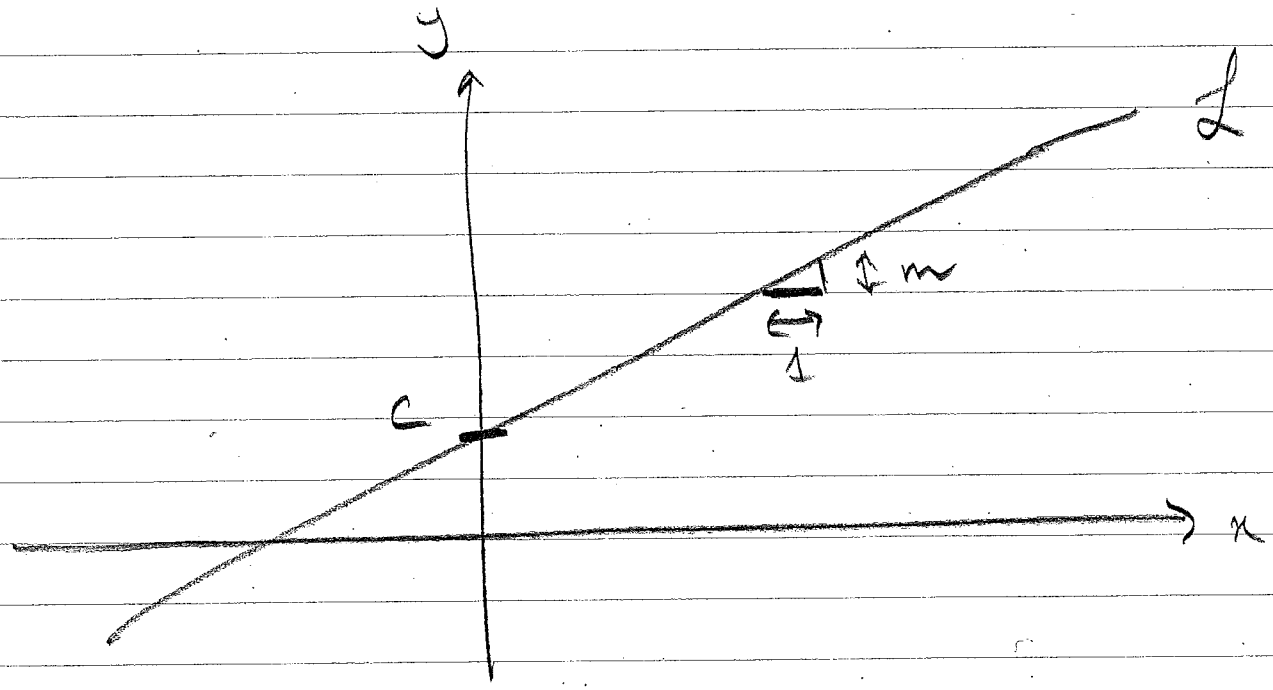
$\Rightarrow x = -5$ or $x = 2$.

Equations of lines continued

Most lines in the xy-plane are described by the equation

$$\boxed{y = mx + c}$$

slope y-intercept



$$\text{Slope} = m = \frac{m}{1} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

can be found given any two points on the line

Example: Find the equation of the line $\mathcal{L}$ passing through $P(2,3)$ and $Q(7,13)$.

Solution: The slope of $\mathcal{L}$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{13-3}{7-2} = \frac{10}{5} = 2,$$

Hence the equation is

$$\boxed{y = 2x + c} \quad \text{for some } c.$$

Put $y = 3$ when $x = 2$, so

$$3 = 2(2) + c = 4 + c,$$

so $c = 3 - 4 = -1$, yielding

$$\boxed{y = 2x - 1}.$$

Check: $3 = 2(2) - 1 = 4 - 1$ ✓

$13 = 2(7) - 1 = 14 - 1$ ✓

Practice: Find the equation of the line $\mathcal{L}$ passing through $(0,1)$ and $(9,-6)$.

Solution: Slope of $\mathcal{L}$ is

$$m = \frac{\text{rise}}{\text{run}} = \frac{-6-1}{9-0} = \frac{-7}{9},$$

so equation is

$$\boxed{y = -\frac{7}{9}x + C} \quad \text{for some } C.$$

But $y=1$ when $x=0$, so

$$1 = -\frac{7}{9}(0) + C = C,$$

so $C=1$, yielding

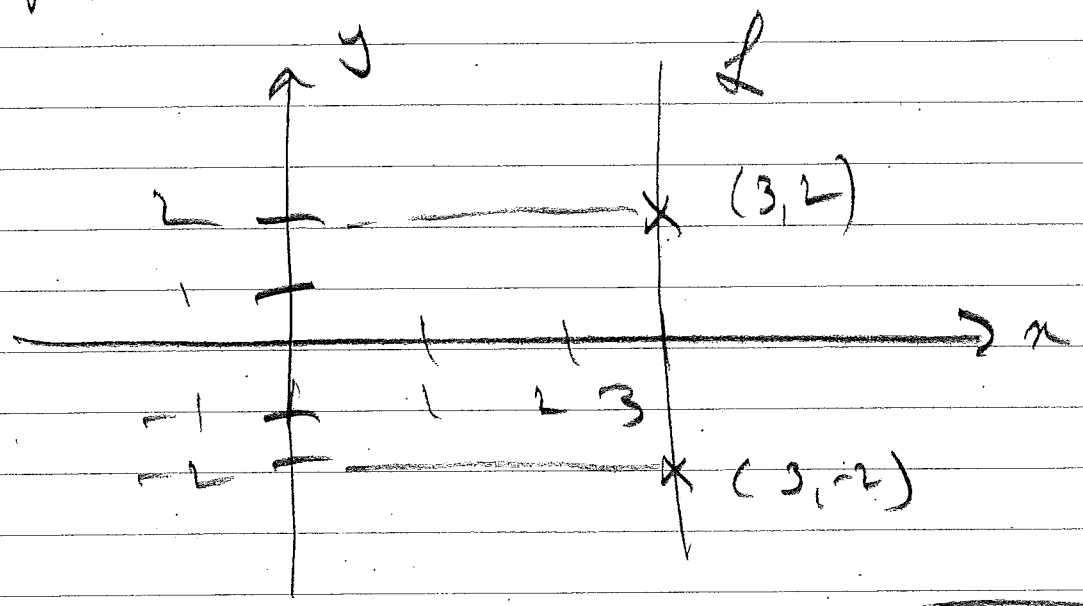
$$\boxed{y = -\frac{7}{9}x + 1}$$

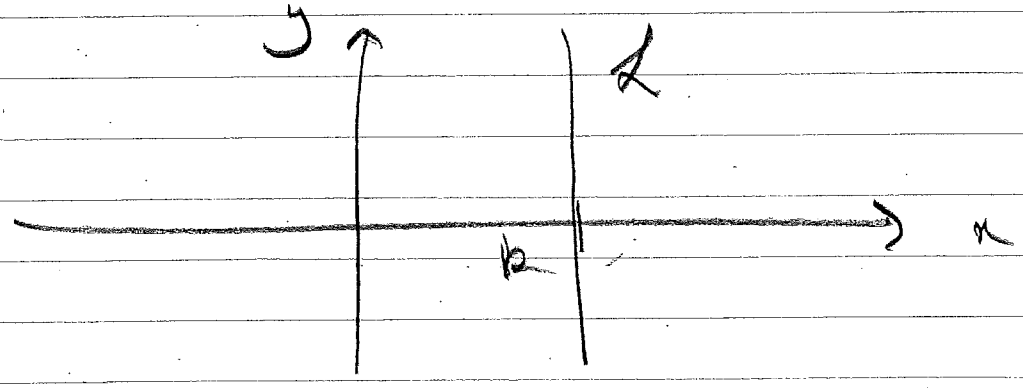
Check: $-\frac{7}{9}(0)+1 = 0+1 = 1$ ✓

$-\frac{7}{9}(9)+1 = -7+1 = -6$ ✓

<u>Warning</u>: some lines are <u>vertical</u>, so have "infinite" slope (so we can't use $y = mx + c$).

<u>Example</u>: Find the equation of the line $\mathcal{L}$ through $(3,2)$ and $(3,-2)$.



<u>Solution</u>: equation of $\mathcal{L}$ is $\boxed{x = 3}$

In general, a vertical line passing through the x-axis at $x = k$ has equation $\boxed{x = k}$