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MATH2022 Lecture 1 Tues 6/3/2018

① Welcome to MATH2022 Linear and Abstract Algebra

- webpage : Google [MATH2022]
- lecturer : David Easdown
- Practice Classes from Week 2 on Mondays
  - links to worksheets from webpage
- Tutorials from Week 2 on Thurs/Fri
  - links to exercise sheets from webpage

preambles with important ideas & useful facts, providing nutshells of course to that point

starred exercises for distinction performance

- First Assignment : due 26 April
  - Q1 routine, but provides a nice explanation of the complex numbers $\mathbb{C}$
  - Q2 application of the Conjugation Principle to solve an elementary but difficult problem in plane geometry.

- Quizzes (multiple choice) on Mondays at 4pm in weeks 5, 9, 13

Please let me know if availability is a problem for you.

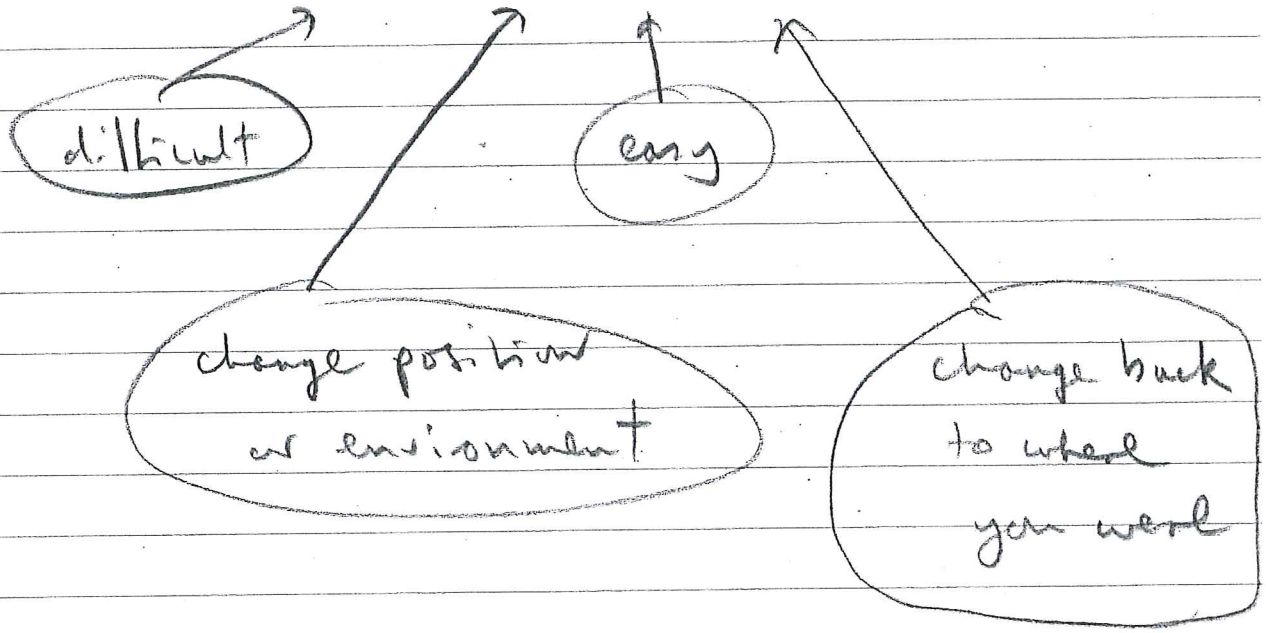
- sample exercises available linked to webpage

② **The Conjugation Principle:**

To do something difficult, change position so that things get easier, and then return.

symbolically:

$$Z = X\,Y\,X^{-1}$$



Examples: poem, cartoon & 15 puzzle

Example: summing a geometric series
(naively using linear algebra)

Put $$M = \begin{bmatrix} 1 & 1 \\ 0 & x \end{bmatrix}.$$

Then $$M^2 = \begin{bmatrix} 1 & 1 \\ 0 & x \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 1 & 1+x \\ 0 & x^2 \end{bmatrix}$$

$$M^3 = \begin{bmatrix} 1 & 1+x \\ 0 & x^2 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & x \end{bmatrix} = \begin{bmatrix} 1 & 1+x+x^2 \\ 0 & x^3 \end{bmatrix}$$

$$\vdots$$

$$M^n = \begin{bmatrix} 1 & 1+x+x^2+\cdots+x^{n-1} \\ 0 & x^n \end{bmatrix}$$

Suppose $|x|<1$ so $\lim_{n\to\infty} x^n = 0$.

Then $$\lim_{n\to\infty} M^n = \begin{bmatrix} 1 & 1+x+x^2+x^3+\cdots \\ 0 & 0 \end{bmatrix}$$

infinite geometric series

How do we find a closed formula for the geometric series? (no tricks)

Answer: "conjugate".

Using eigenvalues & eigenvectors (see later)

$$M = P D P^{-1}$$

where

$$P = \begin{bmatrix} 1 & -\frac{1}{1-x} \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & x \end{bmatrix}$$

Then

$$M^n = \underbrace{(PDP^{-1})(PDP^{-1}) \cdots (PDP^{-1})}_{n \text{ times}}$$

$$= P M^n P^{-1}$$

Hence

$$\lim_{n\to\infty} M^n = \lim_{n\to\infty} \left(P D^n P^{-1}\right)$$

$$= P \left(\lim_{n\to\infty} D^n\right) P^{-1}$$

$$= P \left(\lim_{n\to\infty} \begin{bmatrix} 1 & 0 \\ 0 & x \end{bmatrix}^n\right) P^{-1}$$

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$$= P \left(\lim_{n \to \infty} \begin{bmatrix} 1 & 0 \\ 0 & x^n \end{bmatrix} \right) P^{-1}$$

$$= P \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} P^{-1}$$

$$= \begin{bmatrix} 1 & -\frac{1}{1-x} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{1-x} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{1-x} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{1}{1-x} \\ 0 & 0 \end{bmatrix}.$$

Thus

$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

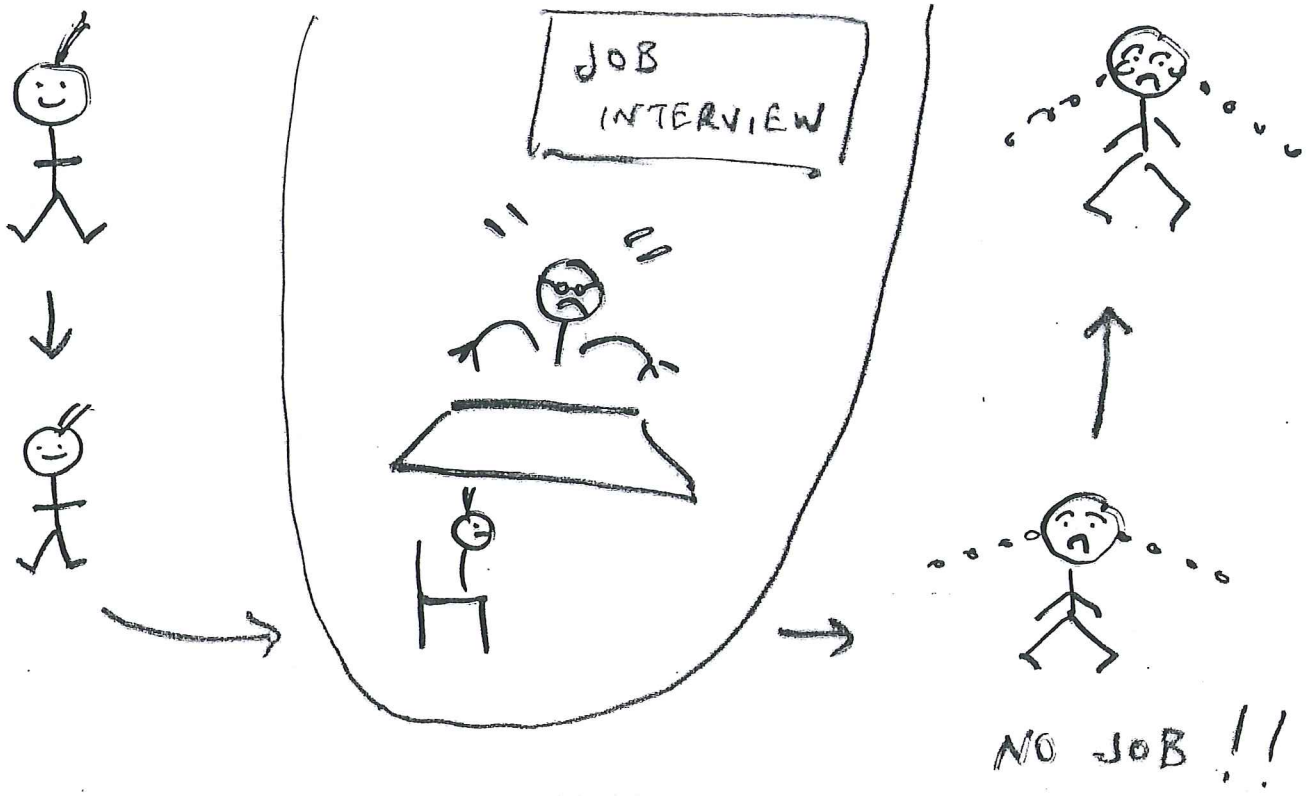
(as expected)

"Avoiding Obstacles"

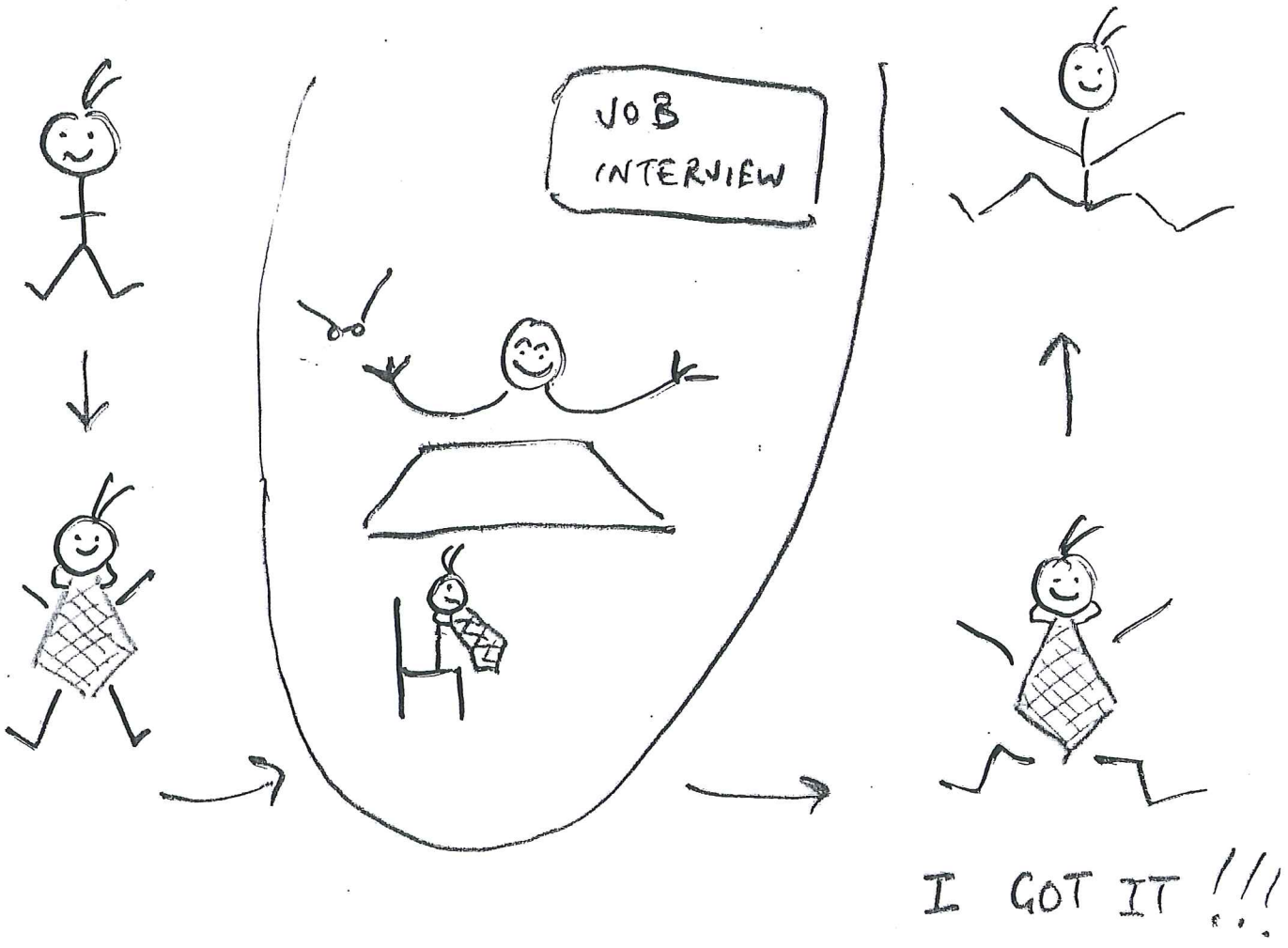
If you want to make a cake
Then find an oven in which it may bake;
But if, on the other hand, you prefer
ice-cream
Then the refrigerator may be the
answer to your dream.

The difference between the foolish and wise
Is sometimes the ability to compromise,
For, if the feelings of others you need
to placate,
The solution may be simply to
conjugate !!

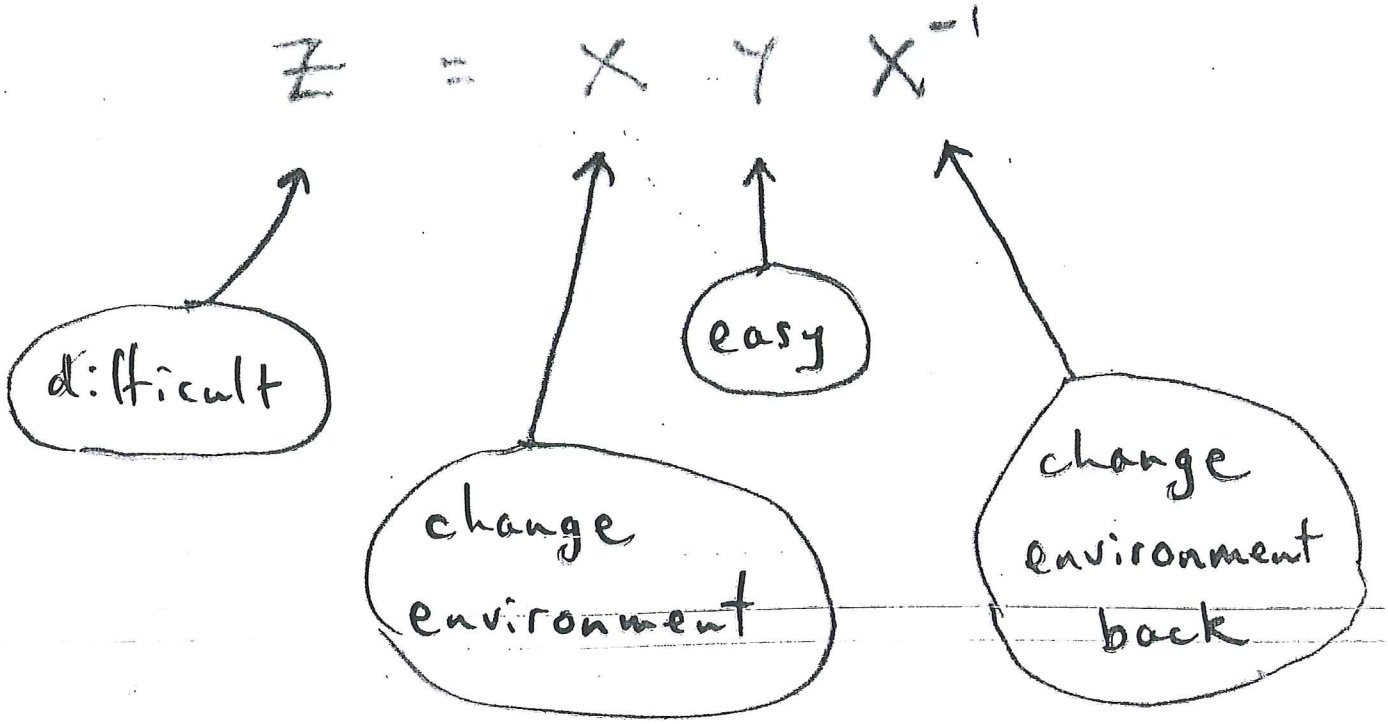
A2.



Using the Conjugation Principle :



The Conjugation Principle :



Examples :

(i) getting a job

(ii) baking a cake

(iii) being transported somewhere

(iv) passing exams (?!)

(v) survival of a chameleon

(vi) using an ATM

(vii) solving puzzles

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Puzzle : put the numbers in order!

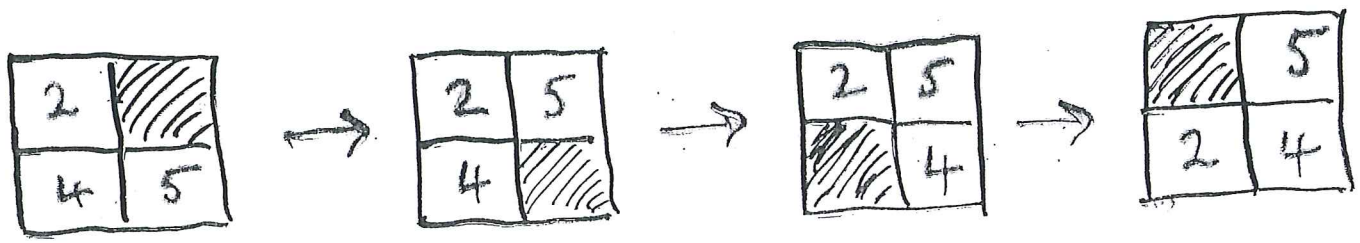
	2	3	1
4	5	6	7
8	9	10	11
12	13	14	15

Solution : "Find an oven in which it may bake"

An oven :

	2
4	5

Cooking :



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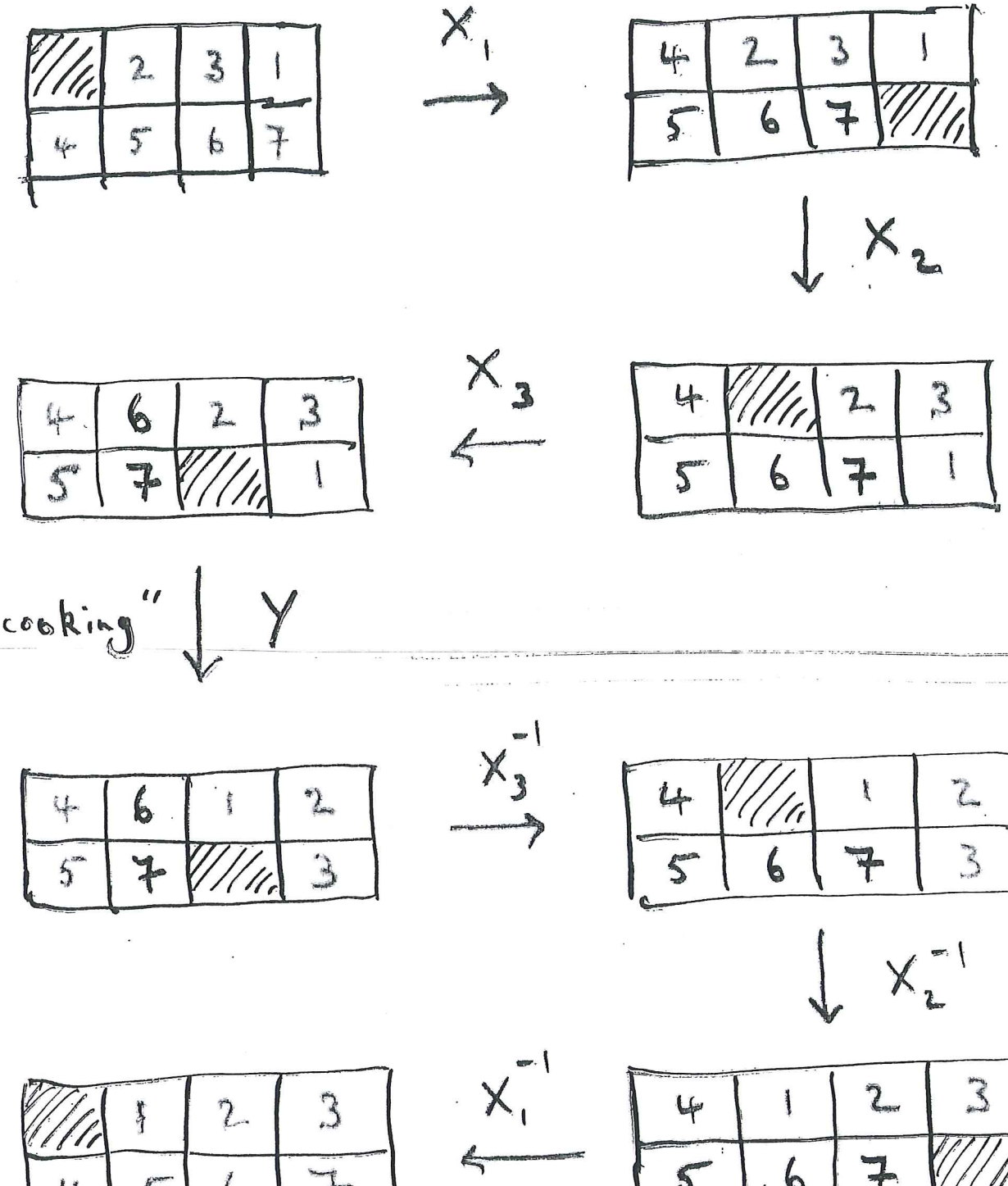

SOLUTION !!

$$Z = X_1 X_2 X_3 \; Y \; (X_1 X_2 X_3)^{-1}$$