The University of Sydney

MATH2022 Linear and Abstract Algebra

Semester 1 **Week 2 Hints and Short Solutions** 2019

1. (a) $\begin{bmatrix} 2 & 4 \\ 0 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -3 \\ -4 & -1 \end{bmatrix}, \begin{bmatrix} 6 & 4 \\ 3 & 6 \end{bmatrix}, \begin{bmatrix} 6 & 10 \\ 9 & 12 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 5 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 5 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} 8 & 5 \\ 4 & 4 \end{bmatrix}$
(d) $\begin{bmatrix} 7 & -1 \\ -4 & 2 \end{bmatrix}, \begin{bmatrix} 0 & 6 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 7 & 12 \\ 9 & 2 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 8 \\ 0 & 9 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ (f) $\begin{bmatrix} 2 & 5 \\ 12 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}$
(g) $\begin{bmatrix} -6 & -3 \\ 4 & 11 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 4 & 4 \end{bmatrix}, \begin{bmatrix} 7 & 10 \\ 4 & 11 \end{bmatrix}$ (h) -3, 4, 10 (i) $\begin{bmatrix} -13 \\ -6 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 12 \end{bmatrix}$
(j) -64, 6, 1

2. Let A be $p \times q$ and B be $r \times s$ and show $p = q = r = s$.

3. Multiply out and simplify. In $\mathbb{Z}_2$, note that $1 + 1 = 0$.

4. Multiply out and simplify using trig identities.

5. Simplify mod 7. Answer should be Wednesday.

6. Simplify mod 24, then calculate mod 7. Answer should be 1 am on Tuesday.

7. (b) Conjecture and then prove $M^n = nM - (n-1)I$.
(c) $\begin{bmatrix} 11 & -5 \\ 20 & -9 \end{bmatrix}, \begin{bmatrix} 21 & -10 \\ 40 & -19 \end{bmatrix}, \begin{bmatrix} 201 & -100 \\ 400 & -199 \end{bmatrix}, \begin{bmatrix} -199 & 100 \\ -400 & 201 \end{bmatrix}$

8. $\mathbb{Z}_7$: $\frac{1}{2} = 4, \frac{1}{3} = 5, \frac{1}{4} = \frac{3}{5} = \frac{5}{6} = 2$. $\mathbb{Z}_8$: $\frac{1}{3} = 3, \frac{3}{5} = 7$. $\mathbb{Z}_9$: $\frac{1}{2} = 5, \frac{1}{4} = 7, \frac{3}{5} = 6$. $\mathbb{Z}_{24}$: $\frac{3}{5} = 15$.

9. $M(M^2) = M(MM) = (MM)M = (M^2)M$

10. $M = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

11. Rows or columns of zeros are inherited by post and pre-multiplication respectively.

12. Both inversion and transposition reverse the order of multiplication.

13. Use associativity and properties of e and inversion.

14. Prove that if at least one matrix is invertible then they all are.

15. Use defining properties of the field, in particular consequences of distributivity and using negatives to cancel common elements on both sides of equations.

16. Use different symbols to denote operations in $\mathbb{Z}_n$ compared with $\mathbb{Z}$, and write out and compare the formal consequences, as expressions in $\mathbb{Z}$, of taking remainders after division by n.