The University of Sydney

MATH2022 Linear and Abstract Algebra

Semester 1 **Week 3 Hints and Short Solutions** 2019

1. Avoid fractions whilst row reducing to find solution $(x, y, z) = (1, 2, -3)$.

2. (a) Unique solution $(0, 0, 0)$ over $\mathbb{R}$, and one parameter solution $\{(4t, t, t) \mid t \in \mathbb{Z}_7\}$ over $\mathbb{Z}_7$.

(b) One parameter solution $\{(-\frac{4t}{3}, 1+\frac{t}{3}, t) \mid t \in \mathbb{R}\}$, simplifying to $\{(t, 1+5t, t) \mid t \in \mathbb{Z}_7\}$ over $\mathbb{Z}_7$.

(c) Two parameter solution $\{(1 - s - \frac{t}{2}, -\frac{1}{2} - \frac{t}{2}, s, t, \frac{3}{2}) \mid s, t \in \mathbb{R}\}$, simplifying to $\{(1 - s + 3t, 3 + 3t, s, t, 5) \mid s, t \in \mathbb{Z}_7\}$ over $\mathbb{Z}_7$.

3. Over $\mathbb{R}$, unique solution when $\lambda \neq 2, -5$, infinite solution when $\lambda = 2$ and no solution when $\lambda = -5$. Over $\mathbb{Z}_5$, unique solution when $\lambda = 1, 3$ or 4, one parameter family of five solutions when $\lambda = 2$, and no solution when $\lambda = 0$.

4. $\alpha = (136425)$, $\beta = (12)(46)$ and $\gamma = (145)(236)$.

5. $\alpha\beta = (134)(25)$, $\alpha\gamma = (165432)$, $\beta\gamma = (1365)(24)$, $\alpha^{-1} = \alpha^5$, $\beta^{-1} = \beta$, $\gamma^{-1} = \gamma^2$.

6. (a) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$, $\begin{bmatrix}0 & 1\\1 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$, $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, $\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$.

(b) First check that $\mathbf{v} \neq \mathbf{0}$ implies $A\mathbf{v} \neq \mathbf{0}$. Use definitions to verify one-one and onto.

(c) Obtain three rotations and three reflections of the triangle.

7. (a) Infinite solution over $\mathbb{R}$, inconsistent over $\mathbb{Z}_5$.

(b) Unique solution over $\mathbb{R}$, inconsistent over $\mathbb{Z}_5$.

(c) Inconsistent over $\mathbb{R}$, finite solution involving two parameters over $\mathbb{Z}_5$

(d) Inconsistent over $\mathbb{R}$, finite solution involving three parameters over $\mathbb{Z}_5$

8. $A = 1$, $B = 3$, $C = 3$, $D = 1$.

9. $p(x) = 4 - 3x + 2x^2 + x^3$ when all conditions hold. Look for parametric families of solutions in each of the cases when (c), (b) and (a) respectively fail.

10. Consider cases $a = 0$ and $a \neq 0$ separately.

11. $\alpha = (123)$, $\beta = (23)$ and use equations to sort α's to the left and β's to the right.

12. Verify that the functions are one-one and onto. Match their composites with elements generated by α and β of the previous exercise and check that there is no collapse.