The University of Sydney

MATH2022 Linear and Abstract Algebra

Semester 1 **Week 4 Hints and Short Solutions** 2019

1. Multiply through by the matrix inverse and deduce that $(x, y, z, w) = (1, -1, -2, -3)$.

2. (a) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$, does not exist, $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. $\det M = 78$, nonzero in $\mathbb{Z}_{11}$, but zero in $\mathbb{Z}_{13}$.

4. $\alpha^{-1} = (6\ 5\ 4\ 3\ 2\ 1)$, $\alpha = (1\ 2)(1\ 3)(1\ 4)(1\ 5)(1\ 6)$, odd;
$\beta^{-1} = (1\ 2)(3\ 4)(8\ 7\ 6\ 5)$, $\beta = (1\ 2)(3\ 4)(5\ 6)(5\ 7)(5\ 8)$, odd;
$\gamma^{-1} = (5\ 3\ 1)(6\ 4\ 2)$, $\gamma = (1\ 3)(1\ 5)(2\ 4)(2\ 6)$, even.

5. (a) $(6\ 5\ 4\ 3\ 2\ 1)$ (b) $(2\ 1)(3\ 6)(4\ 5)$ (c) $(9\ 1)(2\ 3\ 4\ 5)(6\ 8\ 7)$ (d) $(5\ 7\ 3\ 4\ 6\ 2)(1\ 9)$

6. $\alpha = (1\ 2\ 4)$, even; $\beta = (1\ 2\ 3\ 4\ 5)$, even; $\gamma = (1\ 4)(2\ 3\ 5\ 6)$, even; $\delta = (1\ 5\ 3\ 4)(2\ 6\ 7)$, odd; $\varepsilon = (1\ 4\ 7\ 3\ 2\ 5)$, odd.

7. (a) possible (b) impossible (c) possible (d) possible (e) impossible (f) impossible

8. $\begin{bmatrix} 10 & 6 & 5 \\ 2 & 2 & 5 \\ 1 & 3 & 3 \end{bmatrix}$.

9. (a) -14 (b) 0 (c) 32

10. create a row or column of zeros

11. (a) 0 or 1 (b) 0

12. For the first part, think about a cycle decomposition of α; and for the second part, just compose a single transposition, which is odd, with itself to get an even permutation.

13. $1, (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3), (1\,2\,3), (1\,2\,4), (1\,3\,4), (2\,3\,4), (1\,3\,2), (1\,4\,2), (1\,4\,3), (2\,4\,3)$

14. Suppose a subgroup of size 6 exists. Intersect it with the subgroup of size 4 and look for a contradiction in each case where the intersection is trivial and nontrivial.

15. For the last part exploit the fact that $P^{-1}IP = I$ for any invertible matrix P.

16. Set up a matrix equation where $ad - bc$ will become a determinant, and use the relationship between invertibility and the determinant being nonzero.