THE UNIVERSITY OF SYDNEY

MATH2022 LINEAR AND ABSTRACT ALGEBRA

Semester 1 **Week 8 Hints and Short Solutions** 2019

**1.** Addition is unaltered, but, for scalar multiplication, multiply a complex number only by a real number. We have $\mathbb{C} = \langle 1, i \rangle$.

**2. 3.** Only $S_1$ is a subspace. Note that $S_2$ does not contain the zero vector, and closure properties fail for the others.

**4.** Inspection of the reduced row echelon forms reveals that $A$ and $C$ have the same row space, but this differs from the row space of $B$.

**5.** Row reduce $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & -4 & 11 \\ 2 & 4 & -5 & 14 \end{bmatrix}$ to see that the same nonzero rows appear in their reduced row echelon forms.

**6.** (a) (b) Mimic the proofs of corresponding results for groups.

(c) (d) Play with left and right distributivity and exploit negatives.

(e) Play with left distributivity, the fact that $1\mathbf{v} = \mathbf{v}$ and uniqueness of the negative.

(f) Play with the inverse of a nonzero element in the field, the fact that $1\mathbf{v} = \mathbf{v}$ and associativity of scalar multiplication.

**7.** For the converse, express $\mathbf{v} + \mathbf{w} = 1\mathbf{v} + 1\mathbf{w}$ and $\lambda\mathbf{v} = \lambda\mathbf{v} + 0\mathbf{w}$ as linear combinations.

**8.** First show that the zero vector lies in every subspace of a vector space.

**9.** Most properties are inherited directly from the vector space. Need to check existence of the zero vector and negatives.

**10.** The zero vector is the zero function that maps each $x \in X$ to $0 \in F$. The negative of a function $f : X \to F$ maps each $x \in X$ to $-f(x) \in F$.

**11.** Exploit the fact that the transpose of a linear combination of row vectors is the linear combination of the transposes.

**12.** Check that the zero vector lies in every subspace and the set consisting only of the zero vector is a subspace.

**13.** Exploit the fact that the transpose of a linear combination of matrices is the linear combination of the transposes.

**14.** A linear combination of polynomials is a polynomial whose degree does not exceed the maximum degree of the polynomials used to create the linear combination.

**15.** Exploit the triangle inequality applied to a linear combination of bounded functions.