The University of Sydney

MATH2022 Linear and Abstract Algebra

Semester 1 **Week 10 Hints and Short Solutions** 2019

1. (a) $\begin{bmatrix} e^{-t} & 0 \\ 0 & e^{2t} \end{bmatrix}$ (b) $\frac{1}{2}\begin{bmatrix} 1+e^{2t} & -1+e^{2t} \\ -1+e^{2t} & 1+e^{2t} \end{bmatrix}$ (c) $\frac{1}{5}\begin{bmatrix} 2e^{4t}+3e^{-t} & 3e^{4t}-3e^{-t} \\ 2e^{4t}-2e^{-t} & 3e^{4t}+2e^{-t} \end{bmatrix}$

(d) $\begin{bmatrix} 2e^{2t}-e^{-t} & -2e^{2t}+2e^{-t} \\ e^{2t}-e^{-t} & -e^{2t}+2e^{-t} \end{bmatrix}$

2. (a) $x = e^{-t},\ y = 2e^{2t}$ (b) $x = \dfrac{3e^{2t}-1}{2},\ y = \dfrac{3e^{2t}+1}{2}$

(c) $x = \dfrac{8e^{4t}-3e^{-t}}{5},\ y = \dfrac{8e^{4t}+2e^{-t}}{5}$ (d) $x = 3e^{-t}-2e^{2t},\ y = 3e^{-t}-e^{2t}$

3. $A = C = I$, $E = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$, $E^{-1} = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}$

4. (a) $[f]_B^B = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$, $[g]_B^B = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$, $[f]_D^D = \begin{bmatrix} -1 & -3 \\ -4 & -2 \end{bmatrix}$, $[g]_D^D = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(c) $h(x,y) = (3x-y, -x-3y)$, $k(x,y) = (3x-4y, 2x-6y)$

5. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$, $\begin{bmatrix} \frac{1}{2} & -1 & \frac{1}{2} \\ -1 & -1 & 1 \\ \frac{1}{2} & 1 & -\frac{1}{2} \end{bmatrix}$ **6.** $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

7. (a) $\begin{bmatrix} e^{2t} & 0 & 0 \\ 0 & e^{-t} & 0 \\ 0 & 0 & e^{3t} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1-e^{-t} & -1+e^{-t} \\ -1+e^{t} & -1+e^{t}+e^{-t} & 1-e^{-t} \\ -1+e^{t} & -1+e^{t} & 1 \end{bmatrix}$

(c) $\frac{1}{4}\begin{bmatrix} 2e^{-t}+2e^{3t} & -e^{-t}+e^{3t} & -2e^{-t}+2e^{3t} \\ 0 & 4e^{-t} & 0 \\ -2e^{-t}+2e^{3t} & -e^{-t}+e^{3t} & 2e^{-t}+2e^{3t} \end{bmatrix}$

8. (a) $x = -e^{2t}$, $y = -4e^{-t}$, $z = 2e^{3t}$ (b) $x = -7+6e^{-t}$, $y = 7-5e^{t}-6e^{-t}$, $z = 7-5e^{t}$

(c) $x = \dfrac{-e^{-t}-e^{3t}}{2}$, $y = -4e^{-t}$, $z = \dfrac{5e^{-t}-e^{3t}}{2}$

9. (a) $f(x,y,z) = (2x+5y-3z, x-4y+7z)$ (c) $[f]_D^B = \begin{bmatrix} -12 & -41 & -8 \\ 8 & 24 & 5 \end{bmatrix}$

10. $[D]_{B_1}^{B_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$, rank 3, nullity 1. $[D]_{B_2}^{B_2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, rank 2, nullity 0.

$[D]_{B_2}^{B_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$, rank 3, nullity 0.