(2019)

MATH2022 Week 10 Worksheet

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Q1/ What is a **basis** for a vector space?

What is the **dimension** of a vector space?

True or False:

(i) $\{(1,2),(2,1)\}$ is a basis for $\mathbb{R}^2$. T F

(ii) $\{(1,2),(2,1)\}$ is a basis for $\mathbb{Z}_5^2$ T F

(iii) $\{(1,2),(2,1)\}$ is a basis for $\mathbb{Z}_3^2$. T F

(iv) $\{(1,2,3),(4,5,6),(7,8,9)\}$

is a basis for $\mathbb{R}^3$. T F

Hint: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \sim$

Q2/ Let $B = \{(1,2), (2,1)\}$,

$\underset{\sim}{u} = (1,0)$, $\underset{\sim}{v} = (0,1)$, $\underset{\sim}{w} = (1,1)$.

Row reduce

$$\left[\begin{array}{cc|ccc} 1 & 2 & 1 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \end{array}\right] \sim$$

(a) Working over $\mathbb{R}$, find

$[\underset{\sim}{u}]_B =$, $[\underset{\sim}{v}]_B =$

$[\underset{\sim}{w}]_B = [\underset{\sim}{u}]_B + [\underset{\sim}{v}]_B =$

(b) Working over $\mathbb{Z}_5$, find

$[\underset{\sim}{u}]_B =$, $[\underset{\sim}{v}]_B =$

$[\underset{\sim}{w}]_B = [\underset{\sim}{u}]_B + [\underset{\sim}{v}]_B =$

Q3/ Working over $\mathbb{Z}_3$, put

$$M = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix}.$$

(a) Row reduce M:

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 2 \end{bmatrix}$$

(b) Row reduce M^T:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 2 \end{bmatrix}$$

(c) What is the rank of M?

What is the nullity of M?

(d) Find the null space $M^{\perp}$ of M:

(e) Find a basis for $M^{\perp}$:

Q4/ Working over $\mathbb{Z}_5$, put

$$M = \begin{bmatrix} 2 & 3 & 0 & 1 & 1 \\ 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 2 & 3 & 2 \end{bmatrix}.$$

(a) Row reduce M:

$$\begin{bmatrix} 2 & 3 & 0 & 1 & 1 \\ 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 2 & 3 & 2 \end{bmatrix} \sim$$

(b) What is the rank of M? $\boxed{}$

What is the nullity of M? $\boxed{}$

(c) Find the null space $M^{\perp}$ of M:

(d) Find a basis for $M^{\perp}$:

Q5/ Working over $\mathbb{Z}_5$, find the rank and nullity of

$$M = \begin{bmatrix} 1 & 2 & 0 & 1 & 4 \\ 2 & 0 & 1 & 3 & 3 \end{bmatrix}$$

and a basis for $M^{\perp}$.

<u>Solution</u>:

Q6/ (a) Find the following cross-product :

$$\begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 1 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix} =$$

and hence write down the equation of the plane through the origin spanned by

$\underset{\sim}{v} = (1,2,3)$, $\underset{\sim}{w} = (2,-3,4)$:

$\boxed{}$

(b) Row reduce to reduced row echelon form :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 4 \end{bmatrix}$$

and use this to find the span of $\underset{\sim}{v}$ and $\underset{\sim}{w}$:

$\langle \underset{\sim}{v}, \underset{\sim}{w} \rangle =$

Q6/ (c) Use part (b) to deduce the equation of the plane $\mathcal{P}$ through the origin containing $(1,2,3)$ and $(2,-3,4)$:

$$\boxed{}$$

(d) Row reduce the following:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 4 \\ 8 & -5 & 18 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -3 & 4 \\ 3 & 1 & 8 \end{bmatrix} \sim$$

(e) True or False:

(i) $(8,-5,18) \in \mathcal{P}$ T F

(ii) $(3,1,8) \in \mathcal{P}$ T F

Q7/ Row reduce the following:

$$\left[\begin{array}{cc|cc} 1 & 2 & 8 & 3 \\ 2 & -3 & -5 & 1 \\ 3 & 4 & 18 & 8 \end{array}\right] \sim$$

<u>True or False</u>:

(i) $(8, -5, 18)$ is a linear combination of $(1, 2, 3)$ and $(2, -3, 4)$. T F

(ii) $(3, 1, 8)$ is a linear combination of $(1, 2, 3)$ and $(2, -3, 4)$. T F

For the true statement, find the linear combination: