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Information for **MATH2022 Linear and Abstract Algebra**

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### Websites

It is important that you check both the MATH2022 website and the Intermediate Mathematics website regularly:

Intermediate Mathematics webpage: <http://www.maths.usyd.edu.au/u/UG/IM/>

MATH2022 webpage: <http://www.maths.usyd.edu.au/u/UG/IM/MATH2022>

On the MATH2022 page you will find online resources and other useful links, updated regularly by the lecturer.

### Lectures and Practice Classes

Times	Location	Lecturer	Consultation
4 pm Mon (Prac)	to be advised	David Easdown, Carlaw 619	3–4 pm Wed
2 pm Tue	to be advised		
4 pm Wed	to be advised		
3 pm Thu	to be advised		

Lectures run for 13 weeks. The first lecture will be on Tuesday 26 February at 2 pm. The last lecture will be on Thursday 30 May at 3 pm. The first Practice Class will be in Week 2, on Monday 4 March at 4 pm.

### Tutorials and Exercise Sheets

Tutorials (one per week) start in Week 2, currently scheduled for Thursdays and Fridays. You should attend the tutorial given on your personal timetable. Exercise sheets for a given week should be available from the MATH2022 web page. Solutions should be posted on the webpage towards the end of any given week.

### Assessment

Your final raw mark for this unit will be calculated as follows:

- 10%: First Quiz held in Week 5.
- 5%: First Assignment due in Week 7.
- 10%: Second Quiz in Week 9.
- 5%: Second Assignment due in Week 11.
- 10%: Third Quiz in Week 13.
- 60%: Exam at end of semester 1.

There is one examination of two hours duration during the examination period at the end of the semester. Further information about the exam will be made available from the webpage at a later date. Final grades will be returned within one of the following bands: *High Distinction (HD)*, 85–100: representing complete or close to complete mastery of the material; *Distinction (D)*, 75–84: representing excellence, but substantially less than complete mastery; *Credit (CR)*, 65–74: representing a creditable performance that goes beyond routine knowledge and understanding, but less than excellence; *Pass (P)*, 50–64: representing at least routine knowledge and understanding over a spectrum of topics and important ideas and concepts in the course.

## References and notes

Notes will be posted regularly on the MATH2022 website. The following book is available in the Library, can be read online, and relevant to topics in this course:

David Poole. *Linear Algebra A Modern Introduction*.

Poole's book is excellent and the main text used now in First and Second Year linear algebra, and should also be available from the Coop Bookshop.

## Aims and Learning Outcomes

The aim of this unit of study is to introduce, illustrate and formalise some of the most important underlying ideas behind elementary modern linear and abstract algebra and their applications, building on knowledge and experience gained in First Year. By contrast, the development of ideas is more rigorous, general and abstract than the treatment in First Year, and involves more sophisticated mathematical reasoning and an appreciation of some very deep classical theorems. By the end of the semester, students should

- be fluent in analysing and constructing arguments involving matrix arithmetic, permutation and abstract groups, fields and vector spaces;
- understand the definitions, main theorems and corollaries for linearly independent sets, spanning sets, basis and dimension of vector spaces;
- be fluent with linear transformations and operators, and in interpreting, analysing and applying associated abstract phenomena using matrix representations and matrix arithmetic;
- have developed appreciation and strong working knowledge of the theory and applications of elementary permutation groups, their decompositions and relationship to invertible phenomena in linear algebra;
- be fluent with important examples, theorems, algorithms and applications of the theory of inner product spaces, including processes and algorithms involving orthogonality, projections and optimisation.

## Week-by-week outline

Week	Topics
1	Revision of matrix arithmetic in the context of general fields. Introduction to groups.
2	Gaussian elimination in the context of general fields. Permutations and cycle notation.
3	Elementary matrices. Determinants. Conjugation. Even and odd permutations.
4	Eigentheory. Characteristic polynomials. Cayley-Hamilton Theorem. Dihedral groups.
5	Diagonalisation. Stochastic matrices. Perron's Theorem. Subgroups and Cayley's Theorem.
6	Linear transformations of the plane. Product decompositions of groups. Vector spaces.
7	Basis and dimension. Rank-nullity theorem for matrices. Lagrange's Theorem for groups.
8	Linear transformations, operators and matrix representations. Rank-nullity theorem.
9	Jordan forms. Group homomorphisms and Fundamental Homomorphism Theorem.
10	Inner product spaces. Orthogonality. Projections. Direct sum decompositions.
11	Gram-Schmidt process. $QR$ -factorisations. Adjoints. Spectral Theorems.
12	Quadratic forms and diagonalisation. Matrix exponentials and applications.
13	Spectral radius of a matrix and geometric series. Revision.