MATH 2022 Linear and Abstract Algebra

LECTURE 34 Tuesday 21/05/2019

Subspaces, orthonormal bases & projections

- ambient vector space may be "wild"
- a certain subspace may be "tame" or "controlled"

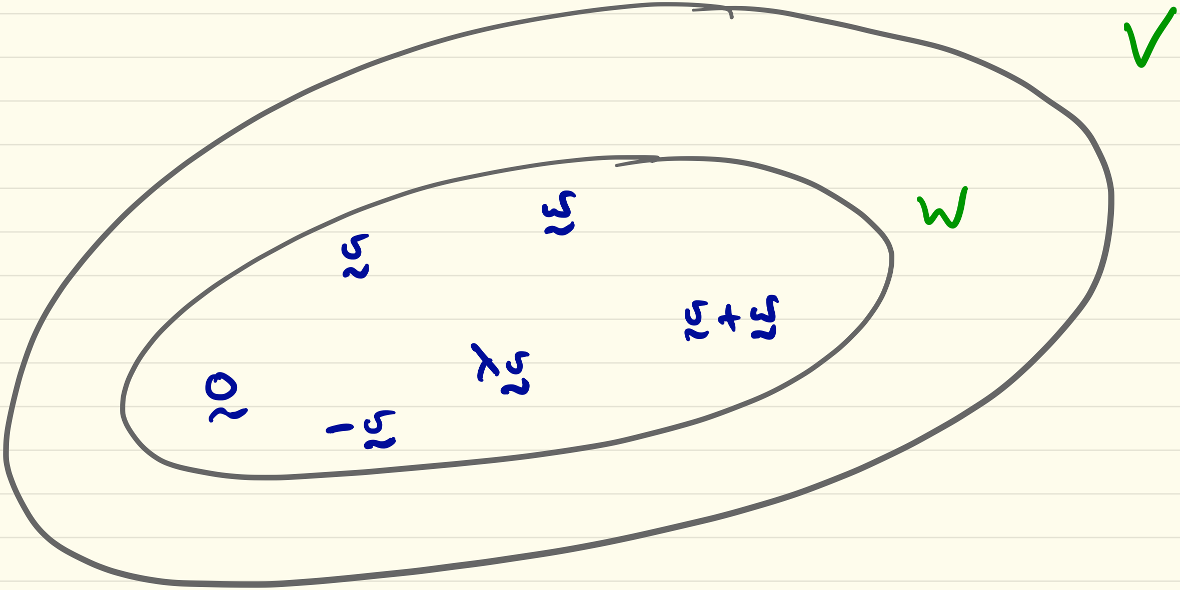
nonempty subset which is a vector space, inheriting addition & scalar multiplication from the ambient vector space

- find or compute an orthonormal basis for the subspace

use this to project arbitrary vectors to "approximate" vectors in the subspace

Subspaces:

Recall that a **subspace** W of a vector space V is a nonempty subset closed under vector space operations.



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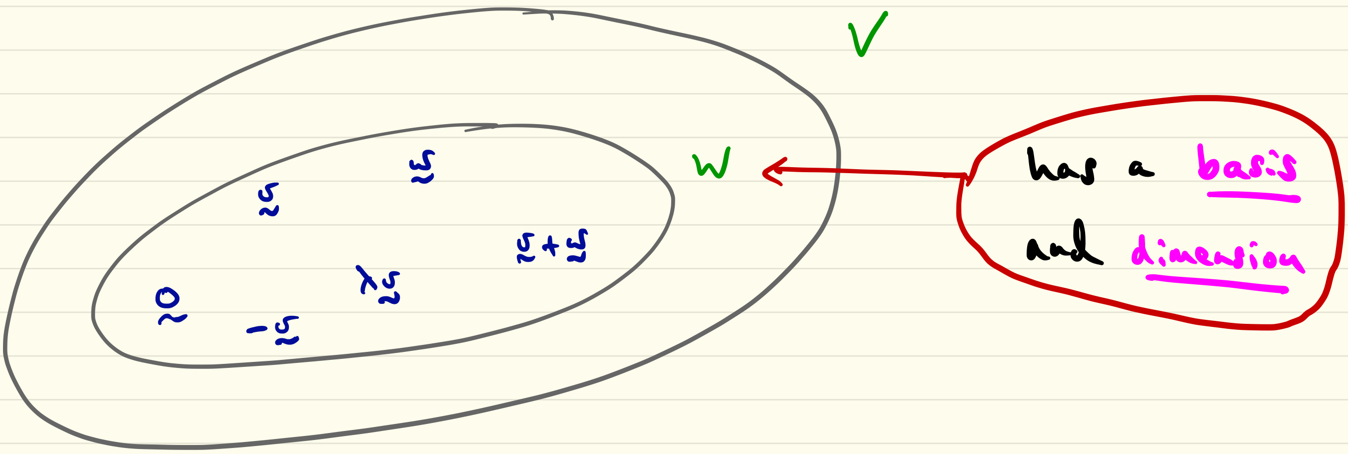

It follows quickly that W contains $\underset{\sim}{0}$, negatives of vectors and satisfies all of the vector space axioms, so is a vector space in its own right.

Theorem: Let W be a subspace of a vector space V.

Then

(i) $\dim W \leq \dim V$,

(ii) $\dim W = \dim V$ iff $W = V$.

Proof: difficult, following from general abstract theory, and beyond the scope of this course. $\square$

Examples:

(a) $\mathbb{R}^2$ has dimension 2, with subspaces

- $\mathbb{R}^2$ of dimension 2,
- $\{(0,0)\} = \langle \phi \rangle$ of dimension 0

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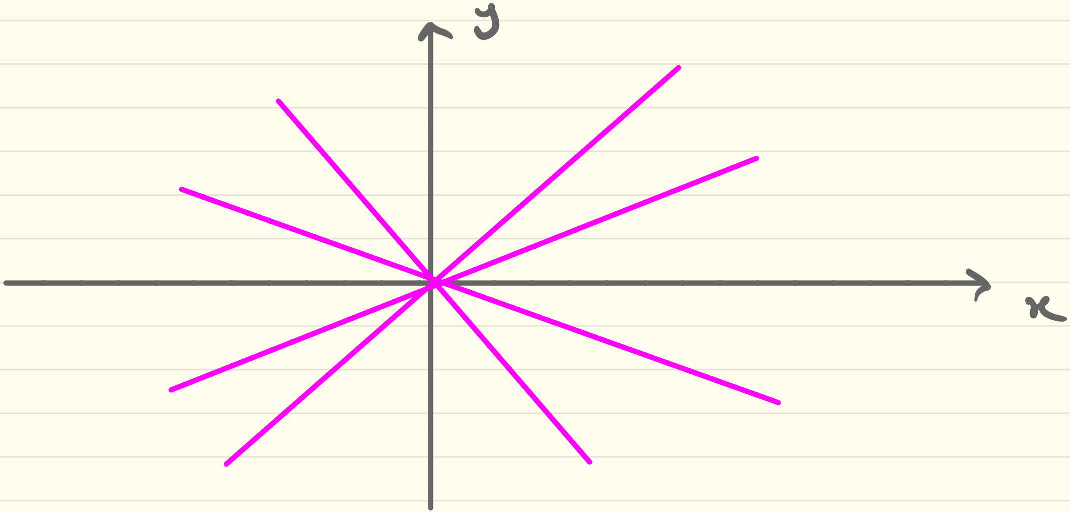
- $\{(0,0)\} = \langle \emptyset \rangle$ of dimension 0
- lines passing through the origin, of dimension 1.



Examples :

(b) $\mathbb{R}^3$ has dimension 3 and its subspaces are $\{(0,0,0)\}$, lines and planes passing through the origin, and $\mathbb{R}^3$, of dimensions 0, 1, 2 and 3 respectively.

(c) $\{$ polynomial functions $\}$ is an infinite dimensional subspace of $\mathbb{R}^{\mathbb{R}} = \{$ functions : $\mathbb{R} \to \mathbb{R}\}$, and $\{1, x, x^2, \dots, x^n, \dots\}$ forms a basis.

(d) A trigonometric polynomial has the form

$$a_0 + a_1 \cos x + a_2 \cos 2x + \cdots + a_n \cos nx$$
$$+ b_1 \sin x + b_2 \sin 2x + \cdots + b_n \sin nx$$

for some $a_0, a_1, \dots, a_n, b_1, \dots, b_n \in \mathbb{R}$.

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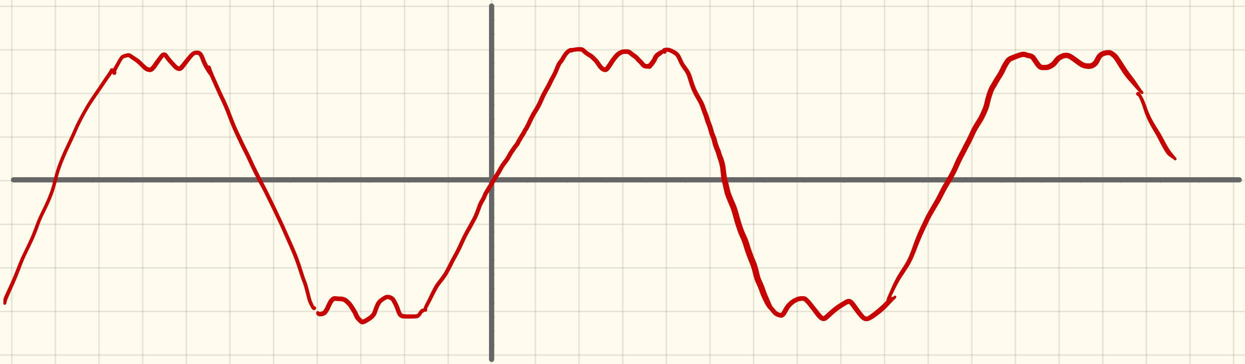
e.g. $f(x) = \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x$

is a trig poly with curve something like



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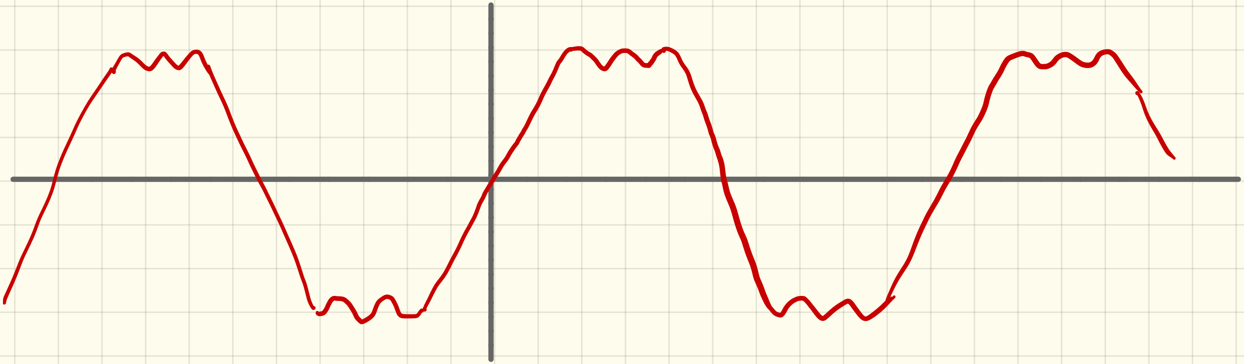
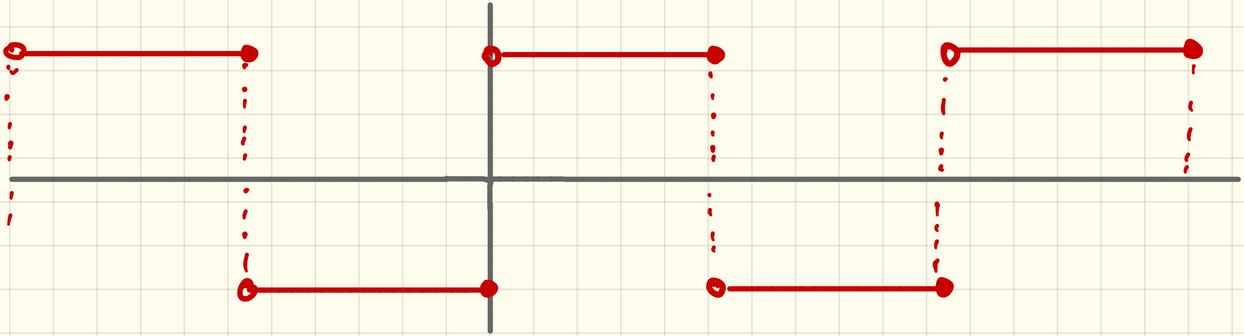


approximating a <u>step function</u>:



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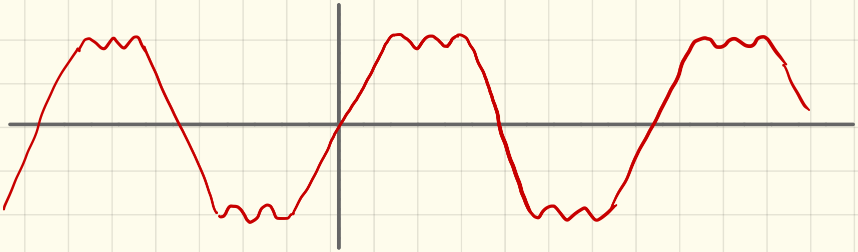

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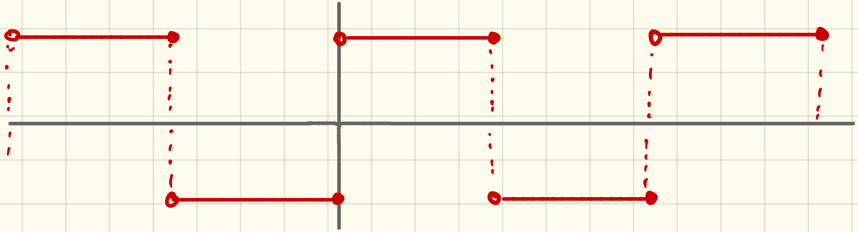


We can get "arbitrarily close" to the step function using the

Fourier series

$$f(x) = \sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \frac{1}{7}\sin 7x + \cdots$$

(analogous to a power series).

The set

$$W = \{ \text{trigonometric polynomials} \}$$

forms an infinite dimensional subspace of the vector space

$$V = \{ \text{periodic functions} : \mathbb{R} \to \mathbb{R} \text{ with period } 2\pi \},$$

and

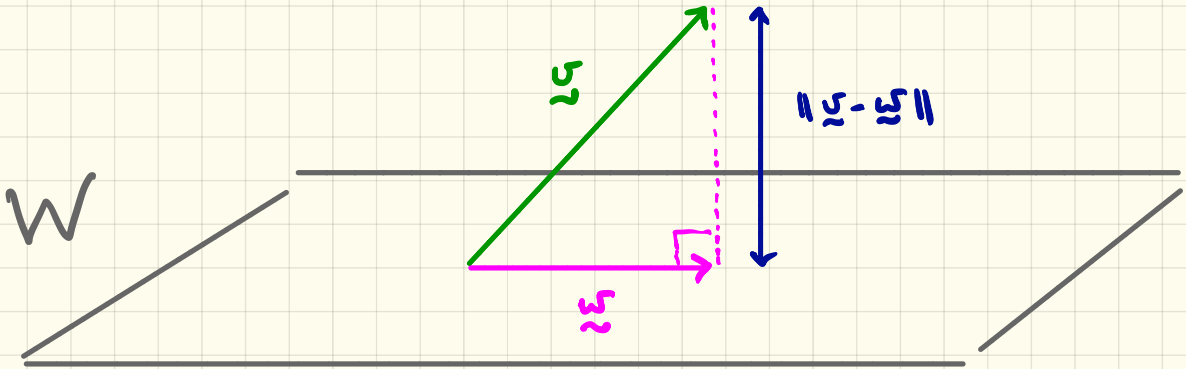
$$\{ 1, \cos x, \cos 2x, \dots, \cos nx, \dots, \\ \sin x, \sin 2x, \dots, \sin nx, \dots \}$$

forms a basis of W (which turns out to be <u>orthogonal</u>).

General problem: Let W be a subspace of an inner product space V. Let $\underset{\sim}{v} \in V$.

Find the vector $\underset{\sim}{w} \in W$ that is **closest** to $\underset{\sim}{v}$, that is, which minimises

$$\| \underset{\sim}{v} - \underset{\sim}{w} \| .$$



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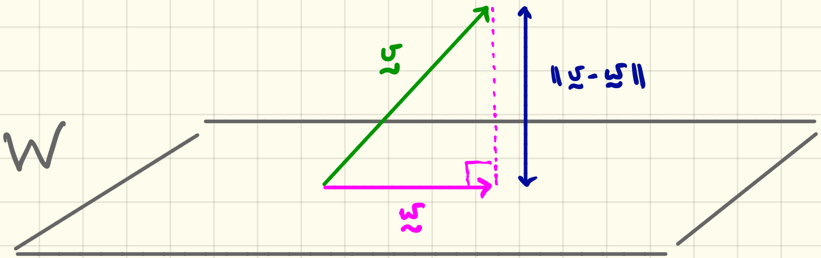

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Step 1: Find an orthonormal basis B for W:

$$B = \{ \underset{\sim}{b}_1, \ldots, \underset{\sim}{b}_k \}$$

such that

(1) $\| \underset{\sim}{b}_i \| = 1$ for $i = 1, \ldots, k$,

(2) $\langle \underset{\sim}{b}_i, \underset{\sim}{b}_j \rangle = 0$ for all $i \neq j$,

(3) $(\forall \underset{\sim}{w} \in W)(\exists \lambda_1, \ldots, \lambda_k \in \mathbb{R})$ $\underset{\sim}{w} = \lambda_1 \underset{\sim}{b}_1 + \ldots + \lambda_k \underset{\sim}{b}_k$.

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Exercise: (2) implies LI

i.e. elements of B have length 1, are pairwise orthogonal and span W

Step 2 : Form the projection of $\underset{\sim}{v}$ onto W

$$\operatorname{proj}_W \underset{\sim}{v} \overset{\text{def}}{=} \langle \underset{\sim}{v}, \underset{\sim}{b}_1 \rangle \underset{\sim}{b}_1 + \langle \underset{\sim}{v}, \underset{\sim}{b}_2 \rangle \underset{\sim}{b}_2 + \cdots + \langle \underset{\sim}{v}, \underset{\sim}{b}_k \rangle \underset{\sim}{b}_k$$

__Theorem__ : $\operatorname{proj}_W \underset{\sim}{v}$ is the closest vector in W to $\underset{\sim}{v}$ and

$$\underset{\sim}{v} - \operatorname{proj}_W \underset{\sim}{v}$$

is orthogonal to everything in W.

(proof later)

Example: Let W be the subspace of $\mathbb{R}^3$ that is the plane given by the equation

$$x - y + z = 0.$$

Let $\underset{\sim}{v} = (1, 2, 3)$. Find the distance from $\underset{\sim}{v}$ to W and the closest point on W to $\underset{\sim}{v}$.

Solution: W is 2-dimensional and spanned by

$$B = \left\{ \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) \right\},$$

which forms an orthonormal basis (and later we explain how to find B).

Put $\quad \underset{\sim}{b}_1 = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right), \quad \underset{\sim}{b}_2 = \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right).$

Then

$$\text{proj}_W \underset{\sim}{v} = (\underset{\sim}{v} \cdot \underset{\sim}{b}_1)\, \underset{\sim}{b}_1 + (\underset{\sim}{v} \cdot \underset{\sim}{b}_2)\, \underset{\sim}{b}_2$$

$$= \left((1,2,3)\cdot\left(\tfrac{1}{\sqrt{2}}, 0, -\tfrac{1}{\sqrt{2}}\right)\right)\left(\tfrac{1}{\sqrt{2}}, 0, -\tfrac{1}{\sqrt{2}}\right) + \left((1,2,3)\cdot\left(\tfrac{1}{\sqrt{6}}, \tfrac{2}{\sqrt{6}}, \tfrac{1}{\sqrt{6}}\right)\right)\left(\tfrac{1}{\sqrt{6}}, \tfrac{2}{\sqrt{6}}, \tfrac{1}{\sqrt{6}}\right)$$

$$= \left(\tfrac{1}{\sqrt{2}} - \tfrac{3}{\sqrt{2}}\right)\left(\tfrac{1}{\sqrt{2}}, 0, -\tfrac{1}{\sqrt{2}}\right) + \left(\tfrac{1}{\sqrt{6}} + \tfrac{4}{\sqrt{6}} + \tfrac{3}{\sqrt{6}}\right)\left(\tfrac{1}{\sqrt{6}}, \tfrac{2}{\sqrt{6}}, \tfrac{1}{\sqrt{6}}\right)$$

$$= -\sqrt{2}\left(\tfrac{1}{\sqrt{2}}, 0, -\tfrac{1}{\sqrt{2}}\right) + \tfrac{8}{\sqrt{6}}\left(\tfrac{1}{\sqrt{6}}, \tfrac{2}{\sqrt{6}}, \tfrac{1}{\sqrt{6}}\right)$$

$$= (-1, 0, 1) + \left(\tfrac{8}{6}, \tfrac{16}{6}, \tfrac{8}{6}\right) = (-1,0,1) + \left(\tfrac{4}{3}, \tfrac{8}{3}, \tfrac{4}{3}\right)$$

$$= \left(\tfrac{1}{3}, \tfrac{8}{3}, \tfrac{7}{3}\right),$$

the closest point on W to $\underset{\sim}{v}$.

Then

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the closest point on W to $\underset{\sim}{v}$.

The distance from $\underset{\sim}{v}$ to W is

$$\|\underset{\sim}{v} - \text{proj}_W \underset{\sim}{v}\| = \|(1,2,3) - (1/3, 8/3, 7/3)\|$$

$$= \|(2/3, -2/3, 2/3)\| = 2/3\, \|(1,-1,1)\| = \tfrac{2}{3}\sqrt{3} = \underline{\underline{\tfrac{2}{\sqrt{3}}}}.$$

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Check orthogonality to W:

$$(\underset{\sim}{v} - \mathrm{proj}_W \underset{\sim}{v}) \cdot \underset{\sim}{b}_1 = \left(\tfrac{2}{3}, -\tfrac{2}{3}, \tfrac{2}{3}\right) \cdot \left(\tfrac{1}{\sqrt{2}}, 0, -\tfrac{1}{\sqrt{2}}\right)$$

$$= \frac{2}{3\sqrt{2}} - \frac{2}{3\sqrt{2}} = 0 \quad \checkmark$$

$$(\underset{\sim}{v} - \mathrm{proj}_W \underset{\sim}{v}) \cdot \underset{\sim}{b}_2 = \left(\tfrac{2}{3}, -\tfrac{2}{3}, \tfrac{2}{3}\right) \cdot \left(\tfrac{1}{\sqrt{6}}, \tfrac{2}{\sqrt{6}}, \tfrac{1}{\sqrt{6}}\right)$$

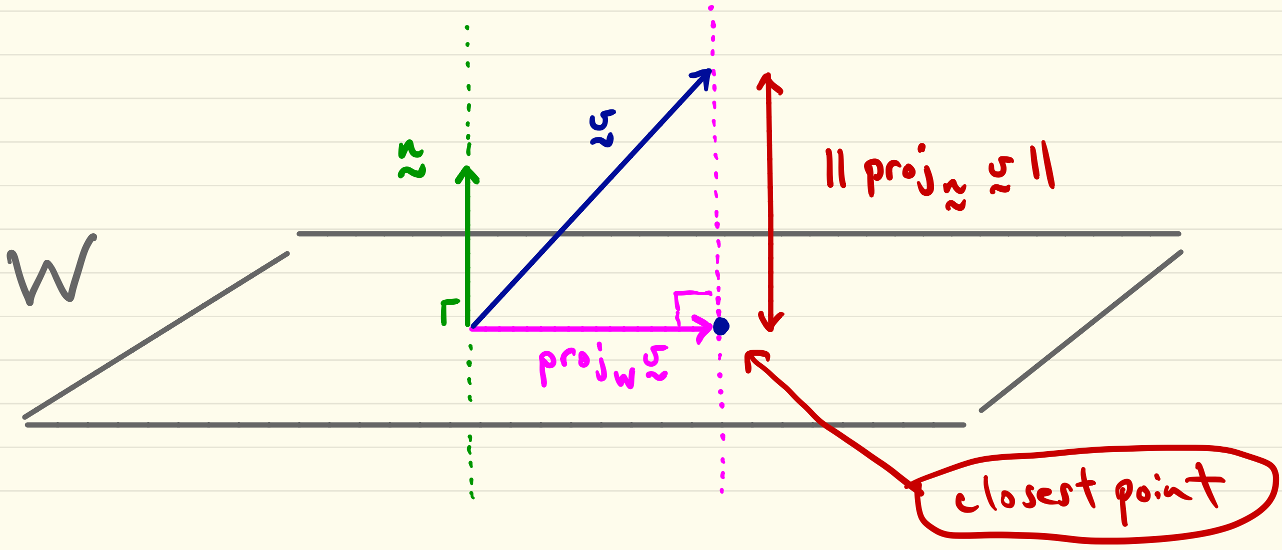
$$= \frac{2-4+2}{3\sqrt{6}} = 0 \quad \checkmark$$

Alternative solution (only works for hyperplanes):

W is given by the equation $x-y+z=0$,

which has normal (perpendicular) vector

$$\underset{\sim}{n} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k} = (1,-1,1).$$



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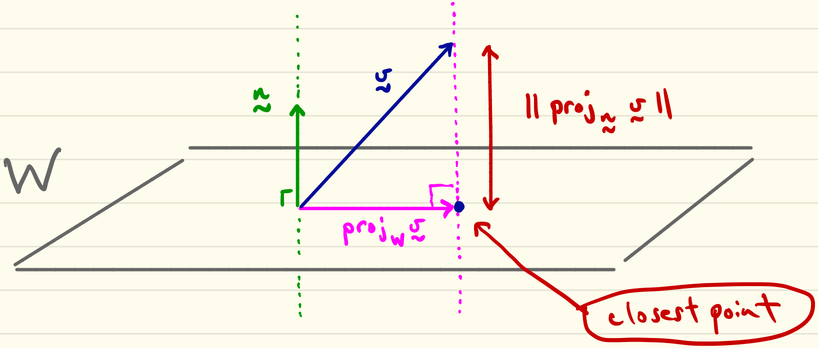

The shortest distance is

$$\|\operatorname{proj}_{\underset{\sim}{n}} \underset{\sim}{v}\| = \left\| \frac{\underset{\sim}{v} \cdot \underset{\sim}{n}}{\|\underset{\sim}{n}\|^2} \underset{\sim}{n} \right\| = \frac{|\underset{\sim}{v} \cdot \underset{\sim}{n}|}{\|\underset{\sim}{n}\|^2} \|\underset{\sim}{n}\| = \frac{|\underset{\sim}{v} \cdot \underset{\sim}{n}|}{\|\underset{\sim}{n}\|}$$

$$= \frac{|(1,2,3) \cdot (1,-1,1)|}{\|(1,-1,1)\|} = \frac{|1-2+3|}{\sqrt{3}} = \frac{2}{\sqrt{3}}.$$

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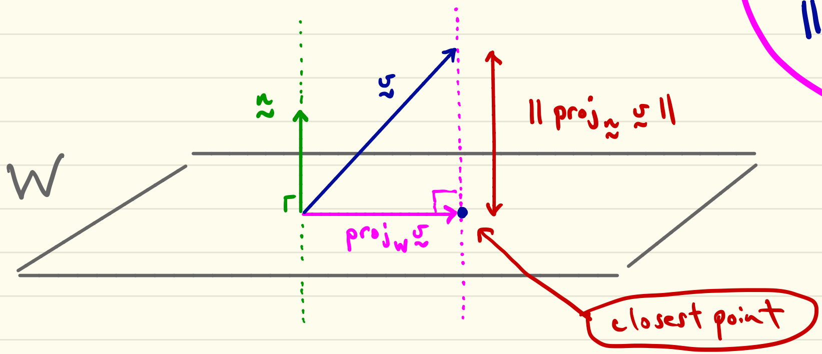

The closest point on W to $\underset{\sim}{v}$ is given by

$$\underset{\sim}{v} - \operatorname{proj}_{\underset{\sim}{n}} \underset{\sim}{v} = (1,2,3) - \frac{\underset{\sim}{v} \cdot \underset{\sim}{n}}{\|\underset{\sim}{n}\|^2} \underset{\sim}{n} = (1,2,3) - \frac{2}{3}(1,-1,1)$$

$$= (1,2,3) - \left(\frac{2}{3}, -\frac{2}{3}, \frac{2}{3}\right) = \left(\frac{1}{3}, \frac{8}{3}, \frac{7}{3}\right)$$

as before.

Third Quiz practice :

Q7/ Which one of the following subsets is not a subspace of the vector space $\mathbb{R}^3$?

(a) $\{ (x,y,z) \in \mathbb{R}^3 \mid x = y = z \}$

(b) $\{ (x,y,z) \in \mathbb{R}^3 \mid x - y + 3z = 0 \}$

(c) $\{ (x,y,z) \in \mathbb{R}^3 \mid x + y + 3z = 2 \}$

(d) $\{ (x,y,z) \in \mathbb{R}^3 \mid x - 2y = 7z \}$

(e) $\{ (x,y,z) \in \mathbb{R}^3 \mid x = 2y ,\ y = z \}$

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solution space of homogeneous system

$$\begin{aligned} x-y \quad &= 0 \\ x \quad -z &= 0 \end{aligned}$$

(a) $\{ (x,y,z) \in \mathbb{R}^3 \mid x=y=z \}$ ✗

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does not contain the zero vector $(0,0,0)$

(d) $\{(x,y,z) \in \mathbb{R}^3 \mid x - 2y = 7z\}$

(e) $\{(x,y,z) \in \mathbb{R}^3 \mid x = 2y, \ y = z\}$

Q8/ Which one of the following subsets does not form a basis for the vector space $\mathbb{Z}_3^3$?

(a) $\{(1,1,0), (1,0,1), (0,2,2)\}$

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dimension 3

Any 3 LI vectors form a basis.

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$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

LI

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LI

X

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(d) $\{(1,0,2), (1,0,1), (0,2,1)\}$

(e) $\{(1,0,0), (1,1,2), (0,2,1)\}$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

LI

$\times$

Q8/ Which one of the following subsets does not form a basis for the vector space $\mathbb{Z}_3^3$?

(a) $\{(1,1,0), (1,0,1), (0,2,2)\}$

(b) $\{(1,2,0), (1,0,1), (0,2,1)\}$

(c) $\{(1,1,1), (1,0,1), (0,2,2)\}$

(d) $\{(1,0,2), (1,0,1), (0,2,1)\}$

(e) $\{(1,0,0), (1,1,2), (0,2,1)\}$

$$\begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

LI

Q8/ Which one of the following subsets does $\underline{\underline{\text{not}}}$ form a basis for the vector space $\mathbb{Z}_3^3$?

(a) $\{(1,1,0), (1,0,1), (0,2,2)\}$

(b) $\{(1,2,0), (1,0,1), (0,2,1)\}$

(c) $\{(1,1,1), (1,0,1), (0,2,2)\}$

(d) $\{(1,0,2), (1,0,1), (0,2,1)\}$

(e) $\{(1,0,0), (1,1,2), (0,2,1)\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

LD

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