1. Avoid fractions whilst row reducing to find solution \((x, y, z) = (1, 2, -3)\).

2. (a) Unique solution \((0, 0, 0)\) over \(\mathbb{R}\), and one parameter solution \(\{(4t, t, t) \mid t \in \mathbb{Z}_7\}\) over \(\mathbb{Z}_7\).
   
   (b) One parameter solution \(\{(-\frac{4t}{3}, 1 + \frac{t}{3}, t) \mid t \in \mathbb{R}\}\), simplifying to \(\{(t, 1 + 5t, t) \mid t \in \mathbb{Z}_7\}\) over \(\mathbb{Z}_7\).
   
   (c) Two parameter solution \(\{(1 - s - \frac{t}{2}, -\frac{1}{2} - \frac{t}{2}, s, t) \mid s, t \in \mathbb{R}\}\), simplifying to \(\{(1 - s + 3t, 3 + 3t, s, t, 5) \mid s, t \in \mathbb{Z}_7\}\) over \(\mathbb{Z}_7\).

3. Over \(\mathbb{R}\), unique solution when \(\lambda \neq 2, -5\), infinite solution when \(\lambda = 2\) and no solution when \(\lambda = -5\). Over \(\mathbb{Z}_5\), unique solution when \(\lambda = 1, 3\) or 4, one parameter family of five solutions when \(\lambda = 2\), and no solution when \(\lambda = 0\).

4. \(\alpha = (136425), \beta = (12)(46)\) and \(\gamma = (145)(236)\).

5. \(\alpha\beta = (134)(25), \alpha\gamma = (165432), \beta\gamma = (1365)(24), \alpha^{-1} = \alpha^5, \beta^{-1} = \beta, \gamma^{-1} = \gamma^2\).

6. (a) \[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}, \begin{bmatrix}
0 & 1 \\
1 & 1
\end{bmatrix}, \begin{bmatrix}
1 & 1 \\
1 & 0
\end{bmatrix}, \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix}, \begin{bmatrix}
1 & 0 \\
1 & 1
\end{bmatrix}, \begin{bmatrix}
1 & 1 \\
0 & 1
\end{bmatrix}.
\]
   
   (b) First check that \(v \neq 0\) implies \(Av \neq 0\). Use definitions to verify one-one and onto.
   
   (c) Obtain three rotations and three reflections of the triangle.

7. (a) Infinite solution over \(\mathbb{R}\), inconsistent over \(\mathbb{Z}_5\).
   
   (b) Unique solution over \(\mathbb{R}\), inconsistent over \(\mathbb{Z}_5\).
   
   (c) Inconsistent over \(\mathbb{R}\), finite solution involving two parameters over \(\mathbb{Z}_5\)
   
   (d) Inconsistent over \(\mathbb{R}\), finite solution involving three parameters over \(\mathbb{Z}_5\)

8. \(A = 1, B = 3, C = 3, D = 1\).

9. \(p(x) = 4 - 3x + 2x^2 + x^3\) when all conditions hold. Look for parametric families of solutions in each of the cases when (c), (b) and (a) respectively fail.

10. Consider cases \(a = 0\) and \(a \neq 0\) separately.

11. \(\alpha = (123), \beta = (23)\) and use equations to sort \(\alpha\)'s to the left and \(\beta\)'s to the right.

12. Verify that the functions are one-one and onto. Match their composites with elements generated by \(\alpha\) and \(\beta\) of the previous exercise and check that there is no collapse.