The University of Sydney

MATH2022 Linear and Abstract Algebra

Semester 1	**Week 11 Hints and Short Solutions**	2020

1. (a) $(-4,-4,8)$ (b) $4\sqrt{6}$ (c) 2 (d) $(2,-2,2)$ (e) $2\sqrt{3}$ (f) $\sqrt{14}+\sqrt{2}$

2. $3,\ \dfrac{\pi}{3}$

3. (a) Check distances between any two vertices are the same. (b) $\cos^{-1}(-\frac{1}{3}) \approx 109.5°$

4. Check dot product of sum and difference of two vectors representing the radius is zero, provided the vectors are different and do not point in opposite directions.

5. $\sqrt{14},\ \sqrt{91},\ \sqrt{105},\ 0,\ \dfrac{\pi}{2}$

6. (a) $\mathbf{i}+2\mathbf{j}$ (b) $\frac{1}{\sqrt{5}}(\mathbf{i}+2\mathbf{j})$ (c) $-\mathbf{i}+5\mathbf{j}$ (d) $\frac{9}{5}\mathbf{i}+\frac{18}{5}\mathbf{j}$ (e) $(\frac{9}{5},\frac{18}{5})$, $\frac{7\sqrt{5}}{5}$

7. (b) $(\frac{11}{6},-\frac{7}{3},-\frac{17}{6})$ (c) $\frac{13\sqrt{6}}{6}$

8. (a) $\|\mathbf{v}\|^2-\frac{(\mathbf{u}\cdot\mathbf{v})^2}{\|\mathbf{u}\|^2}$

9. Routine checking with coordinate-wise operations.

10. Routine checking using properties of the dot product. In $\mathbb{R}^2$, the sum of the squares of the diagonals of a parallelogram add up to the sum of the squares of the lengths of the sides.

11. Use properties of the dot product applied to the square of the length of the sum of two vectors, and then take square roots.

12. Apply the triangle inequality to the expression $(\mathbf{u}-\mathbf{v})+(\mathbf{v}-\mathbf{w})$.

13. Use properties of the inner product and fact that $\langle \mathbf{v},\mathbf{w}\rangle = 0$.

14. Routine checking using properties of the inner product.

15. Routine checking using properties of the definite integral for all but the last inner product axiom. To verify that $\langle f,f\rangle = 0$ implies f is the zero function, argue by contradiction and use continuity of f to find a nonzero area contributing to $\int_{-1}^{1}\big(f(x)\big)^2\,dx$.

16. (a) $\sqrt{2},\ \sqrt{\frac{2}{3}},\ \sqrt{\frac{2}{7}},\ 0,\ 0,\ \frac{2}{5},\ 2\sqrt{\frac{2}{3}}$. The pairs f,g and f,h are orthogonal.

(b) p and q are orthogonal if and only if m and n have different parity.