MATH2022 Week 03

Worksheet

MATH 2022 Week 3 Worksheet

Q1/ The **determinant** of a 2×2 matrix

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is

$$\det M = ad - bc .$$

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 4 \\ -5 & 7 \end{bmatrix}$$

Working over $\mathbb{R}$, find

$\det A =$ $\boxed{}$

$\det B =$ $\boxed{}$

$\det C =$ $\boxed{}$

Which matrix is <u>not</u> invertible? $\boxed{}$

Find

$$A^{-1} =$$

$$C^{-1} =$$

$$AC =$$

$$A^{-1}C^{-1} =$$

$$C^{-1}A^{-1} =$$

$$\det(AC) =$$

$$(AC)^{-1} =$$

$$(CA)^{-1} =$$

Q2/ Put

$$M = \begin{bmatrix} 5 & 5 \\ 5 & -6 \end{bmatrix}.$$

Over $\mathbb{R}$,

$$\det M = \boxed{}$$

and

$$M^{-1} =$$

Over $\mathbb{Z}_7$,

$$M = \begin{bmatrix} 5 & 5 \\ 5 & -6 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -2 & 1 \end{bmatrix},$$

$$\det M = \boxed{}$$

and

$$M^{-1} =$$

Why does M^{-1} not exist over $\mathbb{Z}_{11}$?

$$\boxed{}$$

Q3/ Solve

$$\begin{aligned} x + \quad z &= 0 \\ y + z &= 0 \\ x + y \quad &= 1 \end{aligned}$$

over $\mathbb{R}$:

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{array}\right] \sim$$

over $\mathbb{Z}_3$ by converting previous solution:

Why is there no solution over $\mathbb{Z}_2$?

Q4/ Solve the homogeneous system over $\mathbb{Z}_2$:

$$\begin{aligned} x_1 + x_2 + x_3 + \phantom{x_4 +{}} x_5 &= 0 \\ x_1 + x_2 \phantom{{}+x_3} + x_4 + x_5 &= 0 \\ x_1 + \phantom{x_2 +{}} x_3 \phantom{{}+x_4} + x_5 &= 0 \\ x_1 + x_2 \phantom{{}+x_3} + x_4 \phantom{{}+x_5} &= 0 \end{aligned}$$

after row reducing the following matrix to reduced row echelon form:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix} \sim$$

Q5/ Consider the following permutations of $X = \{1, 2, 3, 4, 5\}$:

$\alpha : 1 \mapsto 2,\ 2 \mapsto 5,\ 3 \mapsto 4,\ 4 \mapsto 1,\ 5 \mapsto 3$

$\beta : 1 \mapsto 3,\ 2 \mapsto 4,\ 3 \mapsto 1,\ 4 \mapsto 5,\ 5 \mapsto 2$

$\gamma : 1 \mapsto 5,\ 2 \mapsto 2,\ 3 \mapsto 4,\ 4 \mapsto 1,\ 5 \mapsto 3$

Express the following using cycle notation:

$\alpha =$

$\beta =$

$\gamma =$

Simplify, using cycle notation:

$\alpha\beta =$

$\beta\alpha =$

$\beta\gamma =$

$\gamma\alpha =$

$\beta\gamma\alpha =$

Q6/ Put $\alpha = (12)(34)$, $\beta = (13)(24)$,

$\gamma = \alpha\beta = (12)(34)(13)(24) = \boxed{}$

Find $\alpha^2 = \beta^2 = \gamma^2 =$

$\beta\gamma =$

$\gamma\alpha =$

and complete the multiplication table:

	1	α	β	γ
1				
α				
β				
γ				

How does this compare with the composition table for symmetries of the rectangle?

The group $G = \{1, \alpha, \beta, \gamma\}$ is abelian? T F

Q7/ Consider polynomials with coefficients from $\mathbb{Z}_2 = \{0,1\}$. Put

$$F = \{0, 1, x, 1+x\}.$$

Complete the addition table for F:

$+$	0	1	x	$1+x$
0				
1				
x				
$1+x$				

Complete the multiplication table for F if

(a) $x^2+1=0$:

$\cdot$	0	1	x	$1+x$
0				
1				
x				
$1+x$				

(b) $x^2+x+1=0$:

$\cdot$	0	1	x	$1+x$
0				
1				
x				
$1+x$				

Which of (a) or (b) yields a field? $\boxed{}$