MATH2022 Week 04 Worksheet

MATH 2022 Week 4 Worksheet

Q1/ Working over $\mathbb{Z}_3 = \{0,1,2\}$, put

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$

Find

$$\det M = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} =$$

Complete the following row reduction to find M^{-1}:

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right] \sim$$

Q2/ Working over $\mathbb{Z}_2 = \{0,1\}$, put

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

Find M^{-1}:

What is $\det M$? $\boxed{}$ (no working required)

Q3/ Working over $\mathbb{Z}_3 = \{0,1,2\}$, identify the e.r.o. in each case, and then find the corresponding elementary matrix:

(a) $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

e.r.o. $\boxed{}$

elementary matrix $E_1 = \boxed{}$

(b) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

e.r.o. $\boxed{}$

elementary matrix $E_2 = \boxed{}$

(c) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

e.r.o. $\boxed{}$

elementary matrix $E_3 = \boxed{}$

(d) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

e.r.o.

elementary matrix $E_4 =$

(e) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e.r.o.

elementary matrix $E_5 =$

(f) $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e.r.o.

elementary matrix $E_6 =$

(g) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

e.r.o.

elementary matrix $E_7 =$

Q4/ Working over $\mathbb{Z}_3$, again put

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$

Consider the elementary matrices from Q3:

$E_1, E_2, E_3, E_4, E_5, E_6, E_7$

Which of these are

lower triangular?

diagonal?

upper triangular?

Express I in terms of M and $E_1, \ldots, E_7$:

$I =$

Now express M in terms of $E_1^{-1}, \ldots, E_7^{-1}$:

$M =$

Find

$E_1^{-1} =$, $E_2^{-1} =$, $E_3^{-1} =$

$E_4^{-1} =$, $E_5^{-1} =$, $E_6^{-1} =$

$E_7^{-1} =$

$E_1^{-1} E_2^{-1} E_3^{-1} =$

$E_4^{-1} E_5^{-1} =$

$E_6^{-1} E_7^{-1} =$

Write down an LDU-decomposition of M:

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} =$$

Q5/ True or false:

(a) $(2\,3\,4) = (2\,3)(3\,4)$ T F

(b) $(2\,3\,4) = (4\,2)(4\,3)$ T F

(c) $(1\,2)(2\,4) = (1\,4)(2\,1)$ T F

(d) $(2\,3\,4)$ is even T F

(e) $(1\,2)(3\,4)(2\,3) = (2\,4\,3\,1)$ T F

(d) $(2\,3\,1\,6\,5)(4\,2) = (6\,5\,1)(2\,3\,1\,4)$ T F

(e) $(2\,3\,1\,6\,5)$ is even T F

(f) $(3\,6\,5)(2\,4\,7)$ is odd T F

(g) $(1\,2)(3\,4)(5\,6)$ is even T F

(h) $(1\,2\,3)(4\,5\,6)(7\,8\,9)$ is odd T F

Q6/ The starting configuration of the 8-puzzle is

1	2	3
4	5	6
7	8	

True or false?

1	2	3
4	6	8
7	5	

is possible T F

2	1	3
4	5	6
7	8	

is possible T F

2	1	3
6	4	5
8	7	

is impossible T F

2	1	4
7		3
8	5	6

is impossible T F