MATH 2022 Week 04 Worksheet
Q1/ Working over $\mathbb{Z}_3 = \{0,1,2\}$, put

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$  

Find

$$\det M = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{vmatrix} =$$

Complete the following row reduction to find $M^{-1}$:

$$\begin{bmatrix} 1 & 1 & 2 & | & 1 & 0 & 0 \\ 2 & 1 & 1 & | & 0 & 1 & 0 \\ 1 & 2 & 1 & | & 0 & 0 & 1 \end{bmatrix} \sim$$
Q2/ Working over $\mathbb{Z}_2 = \{0, 1\}$, put

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$ 

Find $M^{-1}$.

What is $\det M$? □ (no working required)
Q3/ Working over \( \mathbb{Z}_3 = \{0, 1, 2\} \), identify the e.r.o. in each case, and then find the corresponding elementary matrix:

(a) \[
\begin{bmatrix}
1 & 1 & 2 \\
2 & 1 & 1 \\
1 & 2 & 1
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 2 \\
0 & 2 & 0 \\
1 & 2 & 1
\end{bmatrix}
\]
elementary matrix \( E_1 = \)

(b) \[
\begin{bmatrix}
1 & 1 & 2 \\
0 & 2 & 0 \\
1 & 2 & 1
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 2 \\
0 & 2 & 0 \\
0 & 1 & 2
\end{bmatrix}
\]
elementary matrix \( E_2 = \)

(c) \[
\begin{bmatrix}
1 & 1 & 2 \\
0 & 2 & 0 \\
0 & 1 & 2
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 2 \\
0 & 2 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]
elementary matrix \( E_3 = \)
(d) \[
\begin{bmatrix}
1 & 1 & 2 \\
0 & 2 & 0 \\
0 & 0 & 2
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\]
elementary matrix \( E_4 = \)

(e) \[
\begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
elementary matrix \( E_5 = \)

(f) \[
\begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 1 & 2 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]
elementary matrix \( E_6 = \)

(g) \[
\begin{bmatrix}
1 & 0 & 2 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\sim
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
elementary matrix \( E_7 = \)
Q4/ Working over $\mathbb{Z}_3$, again put

$$M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}.$$ 

Consider the elementary matrices from Q3: $E_1, E_2, E_3, E_4, E_5, E_6, E_7$.

Which of these are

- lower triangular?
- diagonal?
- upper triangular?

Express $I$ in terms of $M$ and $E_1, \ldots, E_7$:

$I =$

Nous express $M$ in terms of $E_1^{-1}, \ldots, E_7^{-1}$:

$M =$
Find
\[
E_1^{-1} = ,
E_2^{-1} = ,
E_3^{-1} = ,

E_4^{-1} = ,
E_5^{-1} = ,
E_6^{-1} = 
\]

Write down an LDU-decomposition of \( M \):
\[
M = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} = 
\]
True or false:

(a) \((234) = (23)(34)\)  
(b) \((234) = (42)(43)\)  
(c) \((12)(24) = (14)(21)\)  
(d) \((234)\) is even  
(e) \((12)(34)(23) = (2431)\)  
(f) \((23165)(42) = (651)(234)\)  
(g) \((23165)\) is even  
(h) \((12)(34)(56)\) is even  

True or False:
Q6/ The starting configuration of the 8-puzzle is

```
1 2 3
4 5 6
7 8
```

True or false?

```
1 2 3
4 6 8
7 5
```

is possible T F

```
2 1 3
4 5 6
7 8
```

is possible T F

```
2 1 3
6 4 5
8 7
```

is impossible T F

```
2 1 4
7 3
8 5 6
```

is impossible T F