The University of Sydney

MATH2022 Linear and Abstract Algebra

Semester 1	**Week 9 Hints and Short Solutions**	2018

1. Complex numbers are expressed uniquely as linear combinations of 1 and $i = \sqrt{-1}$.

2. (a) $[\mathbf{v}]_B = \begin{bmatrix} 3 \\ -2 \\ -5 \end{bmatrix}$ (b) $[\mathbf{v}]_B = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ (c) $[\mathbf{v}]_B = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

3. Both have rank 2, and $C = A + 2B$.

4. Rank is 2 and any pair of linearly independent rows and columns generate the row space and column space respectively. Null space is spanned by $\{(1, -2, 3, 0), (-2, 16, 0, 3)\}$.

5. (a) Linearly dependent by inspection.

(b) Linearly independent, seen by row reducing matrix of coefficients.

(c) Linearly independent by inspection.

(d) Linearly dependent using trigonometric identity.

6. Elements of $\mathbb{Q}(\sqrt{2})$ are linear combinations of 1 and $\sqrt{2}$. These are linearly independent because $\sqrt{2}$ is irrational.

7. (a) $[\mathbf{v}]_B = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ (b) $[\mathbf{v}]_B = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$ (c) $[\mathbf{v}]_B = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

8. Use matrices all of whose entries are zero except for one entry which is 1.

9. (a) Rank is 3 over $\mathbb{R}$ and $\mathbb{Z}_3$. Rank is 2, nullspace spanned by $\{(1, 1, 1)\}$, over $\mathbb{Z}_2$.

(b) Rank is 3 over $\mathbb{R}$. Rank is 2, nullspace spanned by $\{(3, 2, 1)\}$, over $\mathbb{Z}_5$.

(c) Rank is 3, nullspace spanned by $\{(1, -2, 1, 1)\}$, over $\mathbb{R}$. Rank is 2, nullspace spanned by $\{(3, 3, 2, 0), (3, 0, 0, 1)\}$, over $\mathbb{Z}_5$.

10. (a) Independent over $\mathbb{R}$ and $\mathbb{Z}_3$, dependent over $\mathbb{Z}_2$.

(b) (c) (d) Independent over $\mathbb{R}$, but dependent over $\mathbb{Z}_5$.

(e) Dependent over both fields.

11. 12. Rearrange equations back and forth.

13. Show that the image of a basis from the domain is a basis for the codomain.

14. If $\mathbf{v}$ is a scalar multiple of $\mathbf{w}$, apply matrix multiplication and compare scalars.

15. If $\mathbf{v}_1$ is a linear combination of $\mathbf{v}_2$ and $\mathbf{v}_3$, apply matrix multiplication and compare scalars, and use the result of the previous exercise.