

MATH2022 Week 06  
Worksheet

Q1/ Put  $M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ .

(a) Find  $M \begin{bmatrix} -1 \\ 1 \end{bmatrix} =$

$$M \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$$

(b) What are the eigenvalues?

(c) Find  $P, P^{-1}$  and diagonal  $D$  such that  $M = PDP^{-1}$ :

$$P = \quad , \quad P^{-1} = \quad , \quad D =$$

(d) Find a formula for  $M^n$ :

$$M^n =$$

(e) Use your formula to find

$$M^3 =$$

$$M^4 =$$

Q2/ Put

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

(a) Find  $M \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} =$

$$M \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} =$$

$$, M \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} =$$

(b) What are the eigenvalues?

(c) Find  $P$  and diagonal  $D$  such that  $M = PDP^{-1}$ :

$$P =$$

$$, D =$$

(d) Find  $P^{-1}$ :

(e) Find  $M^n =$

(f) Thus find  $M^4 =$

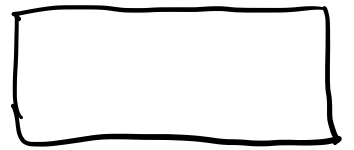
Q3/ Put

$$M = \begin{bmatrix} 5/2 & -1/2 & 0 \\ -1/2 & 5/2 & 0 \\ -1/2 & 1/2 & 2 \end{bmatrix}.$$

(a) Find and factorise

$$\det(\lambda I - M) =$$

(b) What are the eigenvalues?



(c) Find eigenvectors to set up diagonalisation:

Q3/ (continued)

(d) Find  $P$  and diagonal  $D$  such that  $M = PDP^{-1}$ :

$$P = \quad , \quad D =$$

(e) Find  $P^{-1}$ :

(f) Find  $M^n =$

(g) Find  $M^4 =$

Q4/ Put  $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(a) Find surd expressions for the eigenvalues.

(b) What is the largest eigenvalue to 3 d.p.?

$$\lambda_1 = \boxed{\phantom{000000}}$$

(c) What is the smallest eigenvalue to 3 d.p.?

$$\lambda_2 = \boxed{\phantom{000000}}$$

(d) Find  $M^{-1}$ :

Q4/ continued

(e) Put  $\underline{v}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\underline{v}_{k+1} = M \underline{v}_k$  for  $k \geq 0$ ,

$$r_k = \frac{\text{1st entry of } \underline{v}_k}{\text{1st entry of } \underline{v}_{k-1}}, \quad s_k = \frac{\text{2nd entry of } \underline{v}_k}{\text{1st entry of } \underline{v}_k}$$

Complete the following table (to 3 d.p.) :

$\underline{v}_0$	$\underline{v}_1$	$\underline{v}_2$	$\underline{v}_3$	$\underline{v}_4$	$\underline{v}_5$	$\underline{v}_6$	$\underline{v}_7$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 7 \\ 15 \end{bmatrix}$					
$r_k$	1	8					
$s_k$	3	2.143					

find  $M \begin{bmatrix} 1 \\ s_7 \end{bmatrix} =$

What do you notice?

Q4/ continued

(f) Put  $\underline{w}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\underline{w}_{k+1} = M^{-1} \underline{w}_k = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix} \underline{w}_k$ .

$t_k = \frac{\text{1st entry of } \underline{w}_k}{\text{1st entry of } \underline{w}_{k-1}}$ ,  $u_k = \frac{\text{2nd entry of } \underline{w}_k}{\text{1st entry of } \underline{w}_k}$

Complete the following table (to 3 d.p.) :

$\underline{w}_0$	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -2 \\ 1.5 \end{bmatrix}$	$\begin{bmatrix} 5.5 \\ -3.75 \end{bmatrix}$			
$t_k$	-2				
$u_k$	-0.75				

Find  $M \begin{bmatrix} 1 \\ u_5 \end{bmatrix} =$

What do you notice?



Q5/ Let  $G$  be a cyclic group with 8 elements, expressed multiplicatively, say

$$G = \langle a \rangle = \{a, a^2, a^3, a^4, a^5, a^6, a^7, a^8 = 1\}.$$

Find

$$\langle a^2 \rangle =$$

$$\langle a^3 \rangle =$$

$$\langle a^4 \rangle =$$

$$\langle a^5 \rangle =$$

$$\langle a^6 \rangle =$$

$$\langle a^7 \rangle =$$

For which  $i \in \{0, 1, \dots, 7\}$  is it the case that  $\langle a^i \rangle = G$  ?

$i =$