MATH2022 Week 06

Worksheet

MATH 2022 Week 6 Worksheet

Q1/ Put $M = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

(a) Find $M \begin{bmatrix} -1 \\ 1 \end{bmatrix} =$

$M \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$

(b) What are the eigenvalues? $\boxed{}$

(c) Find P, P^{-1} and diagonal D such that $M = PDP^{-1}$:

$P =$, $P^{-1} =$, $D =$

(d) Find a formula for M^n:

$M^n =$

(e) Use your formula to find

$M^3 =$, $M^4 =$

Q2/ Put

$$M = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

(a) Find $M \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} =$

$M \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} =$, $M \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} =$

(b) What are the eigenvalues? $\boxed{}$

(c) Find P and diagonal D such that $M = PDP^{-1}$:

$P =$, $D =$

(d) Find P^{-1}:

(e) Find $M^n =$

(f) Thus find $M^4 =$

Q3/ Put

$$M = \begin{bmatrix} 5/2 & -1/2 & 0 \\ -1/2 & 5/2 & 0 \\ -1/2 & 1/2 & 2 \end{bmatrix}.$$

(a) Find and factorise

$\det(\lambda I - M) =$

(b) What are the eigenvalues? $\boxed{}$

(c) Find eigenvectors to set up diagonalisation:

Q3/ (continued)

(d) Find P and diagonal D such that $M = PDP^{-1}$:

$P =$, $D =$

(e) Find P^{-1}:

(f) Find $M^n =$

(g) Find $M^4 =$

Q4/ Put $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(a) Find surd expressions for the eigenvalues.

(b) What is the largest eigenvalue to 3 d.p.?

$$\lambda_1 = \boxed{\qquad\qquad}$$

(c) What is the smallest eigenvalue to 3 d.p.?

$$\lambda_2 = \boxed{\qquad\qquad}$$

(d) Find M^{-1}:

Q4/ continued

(e) Put $\underset{\sim}{v}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\underset{\sim}{v}_{k+1} = M \underset{\sim}{v}_k$ for $k \geq 0$,

$$r_k = \frac{\text{1st entry of } \underset{\sim}{v}_k}{\text{1st entry of } \underset{\sim}{v}_{k-1}}, \quad s_k = \frac{\text{2nd entry of } \underset{\sim}{v}_k}{\text{1st entry of } \underset{\sim}{v}_k}$$

Complete the following table (to 3 d.p.):

$\underset{\sim}{v}_0$	$\underset{\sim}{v}_1$	$\underset{\sim}{v}_2$	$\underset{\sim}{v}_3$	$\underset{\sim}{v}_4$	$\underset{\sim}{v}_5$	$\underset{\sim}{v}_6$	$\underset{\sim}{v}_7$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$	$\begin{bmatrix} 7 \\ 15 \end{bmatrix}$					
r_k	1	8					
s_k	3	2.143					

Find $M \begin{bmatrix} 1 \\ s_7 \end{bmatrix} =$

What do you notice?

Q4/ continued

(f) Put $\underset{\sim}{w}_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\underset{\sim}{w}_{k+1} = M^{-1}\underset{\sim}{w}_k = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix} \underset{\sim}{w}_k$.

$$t_k = \frac{\text{1st entry of } \underset{\sim}{w}_k}{\text{1st entry of } \underset{\sim}{w}_{k-1}}, \quad u_k = \frac{\text{2nd entry of } \underset{\sim}{w}_k}{\text{1st entry of } \underset{\sim}{w}_k}$$

Complete the following table (to 3 d.p.):

$\underset{\sim}{w}_0$	$\underset{\sim}{w}_1$	$\underset{\sim}{w}_2$	$\underset{\sim}{w}_3$	$\underset{\sim}{w}_4$	$\underset{\sim}{w}_5$
$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -2 \\ 1.5 \end{bmatrix}$	$\begin{bmatrix} 5.5 \\ -3.75 \end{bmatrix}$			
t_k	-2				
u_k	-0.75				

Find $M \begin{bmatrix} 1 \\ u_5 \end{bmatrix} =$

What do you notice?

Q5/ Let G be a cyclic group with 8 elements, expressed multiplicatively, say

$$G = \langle a \rangle = \{ a, a^2, a^3, a^4, a^5, a^6, a^7, a^8 = 1 \}.$$

Find

$\langle a^2 \rangle =$

$\langle a^3 \rangle =$

$\langle a^4 \rangle =$

$\langle a^5 \rangle =$

$\langle a^6 \rangle =$

$\langle a^7 \rangle =$

For which $i \in \{0, 1, \dots, 7\}$ is it the case that $\langle a^i \rangle = G$?

$\boxed{i =}$