MATH2022 Week 07

Worksheet

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Q1/ Solve the following system working over $\mathbb{Z}_3$:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 + x_5 &= 0 \\ x_1 \qquad\quad + x_3 \qquad\quad + x_5 &= 1 \\ x_1 + 2x_2 + x_3 + 2x_4 + x_5 &= 2 \\ 2x_2 \qquad\quad + 2x_4 \qquad &= 1 \end{aligned}$$

What is the size of the solution set?

$\boxed{}$

Q2/ Consider the following matrices where $\theta \in \mathbb{R}$:

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \quad T_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$

Simplify the following:

(a) $T_\theta T_\varphi = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{bmatrix}$

$=$

(b) $T_{\pi/2} T_{\pi/3} =$

(c) $R_{\pi/3} T_{\pi/2} R_{\pi/3} =$

(d) $T_{\pi/3} R_{\pi/2} T_{\pi/3} =$

(e) $R_{2\pi/3} T_{\pi/2} R_{\pi/4} T_{\pi/3}$

$=$

Q3/ Put $M = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$,

$A = \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Find elementary matrices E_1, E_2, E_3 such that

(a) $E_1 M = A$, $\boxed{E_1 = \quad\quad\quad}$

(b) $E_2 A = B$, $\boxed{E_2 = \quad\quad\quad}$

(c) $E_3 B = I$, $\boxed{E_3 = \quad\quad\quad}$

Express M as a product of elementary matrices :

Q4/ Let M be the following stochastic matrix

$$M = \begin{bmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{bmatrix}.$$

(a) Row reduce $I-M$ and hence find the eigenspace corresponding to the eigenvalue 1:

$I-M=$

(b) Now find the steady state probability vector $\underset{\sim}{s}$ for M:

Q5/ Let M be the following stochastic matrix

$$M = \begin{bmatrix} 0.3 & 0 & 0.2 \\ 0.3 & 0.5 & 0.3 \\ 0.4 & 0.5 & 0.5 \end{bmatrix}$$

(a) Row reduce $I-M$ and hence find the steady state probability vector $\underset{\sim}{s}$ for M:

$I - M =$

Q6/ Consider

$$M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

(a) Why is M doubly stochastic?

(b) Write down immediately a steady state probability vector $\underset{\sim}{v}$ for M:

$$\underset{\sim}{v} = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

(c) Let $\underset{\sim}{v}_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\underset{\sim}{v}_{k+1} = M \underset{\sim}{v}_k$ for $k \geq 0$. Explain why $\lim_{n \to \infty} M^n \underset{\sim}{v}_0$ does not exist:

Q7/ In each case find all z such that

$$\mathbb{Z}_n = \langle z \rangle$$

with respect to addition.

(a) $\mathbb{Z}_3 = \langle z \rangle$ where

$z = \boxed{}$

(b) $\mathbb{Z}_4 = \langle z \rangle$ where

$z = \boxed{}$

(c) $\mathbb{Z}_5 = \langle z \rangle$ where

$z = \boxed{}$

(d) $\mathbb{Z}_8 = \langle z \rangle$ where

$z = \boxed{}$

(e) $\mathbb{Z}_9 = \langle z \rangle$ where

$z = \boxed{}$

Q8/ Working over $\mathbb{Z}_3 = \{0, 1, -1\}$, complete the following multiplication table mod x^2+1 (that is, $x^2+1=0$):

	0	1	-1	x	$1+x$	$-1+x$	$-x$	$1-x$	$-1-x$
0									
1									
-1									
x									
$1+x$									
$-1+x$									
$-x$									
$1-x$									
$-1-x$									

Do we get a field? $\boxed{}$

The nonzero elements form a cyclic group. Find all of its generators: $\boxed{}$