

**Attachment to Examination Paper (June 2003 )**  
**MATH2011 TOPICS IN DISCRETE MATHEMATICS**

**Name:**

**Student No.:**

**Section A**

Write your answers to this section in the places indicated.

Place this completed sheet inside the answer booklet for Section B.

- A1.** Complete the following table giving the number of selections of  $m$  objects from a set of size  $n$  in the various cases:

selection	ordered	unordered
with repetition		
without repetition		

Find the following numbers, expressing each final answer as a single integer.  
 (Include brief working if you wish.)

- (i) The number of ordered sequences of 3 digits where the first digit is not 0 but repetitions are allowed:

**Answer:**

- (ii) The number of ordered sequences of 3 digits without repetition where the first digit is not 0:

**Answer:**

- (iii) The number of ways, unordered without repetition, of choosing 3 balls from 10 balls:

**Answer:**

- (iv) The number of ways, unordered with repetition, of choosing 3 balls from an unlimited supply of balls in 10 different colours:

**Answer:**

[10 marks]

**SEE OVER FOR QUESTION A2**

**A2.** Consider each of the following statements. Circle **T** if you believe the statement is true. Circle **F** if you believe the statement is false. (Simple guessing is inadvisable. Marks may be deducted for more than three incorrect answers.)

- (i) If  $A$  and  $B$  are finite sets then  $|A \cap B| = |A| + |B| - |A \cup B|$ . **T F**
- (ii) If there are 80 students in the class then at least 4 have surnames beginning with the same letter. **T F**
- (iii) If  $A, B, C$  are any sets then  $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ . **T F**
- (iv) There exist sets  $A, B$  and  $C$  such that  $|A| = 5, |B| = 4, |C| = 3, |A \cup B \cup C| = 10, |A \cap B| = 2$  and  $|A \cap B \cap C| = 1$ . **T F**
- (v) To say that a set  $X$  of integers has a largest element is to say  $(\forall x \in X)(\exists m \in X) \quad x \leq m$ . **T F**
- (vi) The negation of  $(\forall x)(\exists y) P(x, y) \Rightarrow Q(x, y)$  is  $(\exists x)(\forall y) P(x, y) \wedge \sim Q(x, y)$ . **T F**
- (vii) MERGESORT is a divide and conquer algorithm. **T F**
- (viii) BUBBLESORT has running time  $O(N \log N)$ . **T F**
- (ix) If  $N$  is any integer then  $N = \lfloor N/2 \rfloor + \lceil N/2 \rceil$ . **T F**
- (x) The Euclidean Algorithm is a divide and conquer algorithm. **T F**
- (xi) The g.c.d. of two consecutive Fibonacci numbers is 1. **T F**
- (xii) The equation  $4x = 3 \pmod{9}$  has no integer solution for  $x$ . **T F**
- (xiii) The system of equations  $\begin{cases} x = 0 \pmod{8} \\ x = 10 \pmod{7} \end{cases}$  has a unique solution for  $x \in \mathbb{Z}_{56}$ . **T F**
- (xiv) Fermat's Little Theorem implies that if  $x$  is an integer not divisible by a prime  $p$  then  $x^p = x \pmod{p}$ . **T F**
- (xv) A recurrence relation  $a_n = ra_{n-1} + sa_{n-2} + f(n)$  is homogeneous if  $f(n) \neq 0$ . **T F**
- (xvi) If  $c_n$  is the complementary function and  $p_n$  is a particular solution of a recurrence  $a_n = ra_{n-1} + sa_{n-2} + f(n)$ , then the general solution is  $a_n = c_n + p_n$ . **T F**
- (xvii) The sequence  $0, 1, 2, 3, \dots$  has generating function  $\sum_{n=0}^{\infty} nz^{n+1}$ . **T F**
- (xviii) If  $\sum_{n=0}^{\infty} a_n z^n = \frac{1}{1 - 3z + 2z^2}$  then  $a_2 = 9$ . **T F**
- (xix) If  $\det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{bmatrix} > 0$  and  $P = (x_1, x_2), Q = (y_1, y_2), R = (z_1, z_2)$ , then the triangle  $\triangle PQR$  is oriented clockwise. **T F**
- (xx)  $(3, 5)$  lies outside the triangle with vertices  $(0, 0), (4, 0), (2, 7)$ . **T F**

[10 marks]