Attachment to Examination Paper (June 2004)

MATH2011 TOPICS IN DISCRETE MATHEMATICS

Name:

Student No.:

Section A

Write your answers to this section in the places indicated.

Place this completed sheet inside the answer booklet for Section B.

A1. Complete the following truth tables:

P	Q	$P \vee Q$	$P \wedge Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
T	T				
T	F				
F	T				
F	F				

P	Q	$\sim (P \vee Q)$	$\sim P \wedge \sim Q$	$\sim (P \vee Q) \iff \sim P \wedge \sim Q$
T	T			
T	F			
F	T			
F	F			

(i) Write down the *converse* of $P \Rightarrow Q$? **Answer:**

(ii) Write down the *contrapositive* of $P \Rightarrow Q$? **Answer:**

(iii) Find an expression which is logically equivalent to $\sim (\sim P \wedge Q)$ but does not use the negation symbol.

Answer:

[10 marks]

SEE OVER FOR QUESTION A2

A2. Consider each of the following statements. Circle **T** if you believe the statement is true. Circle **F** if you believe the statement is false. (Simple guessing is inadvisable. Marks may be deducted for more than three incorrect answers.)

(i) The integer 221 is a prime number. **T** **F**

(ii) If $\lim_{N\to\infty} \dfrac{g(N)}{f(N)} = \infty$ and $\lim_{N\to\infty} \dfrac{g(N)}{h(N)} = 0$ then $f(N) = O(h(N))$. **T** **F**

(iii) If A, B, C are any sets then $C\backslash(A \cup B) = (C\backslash A) \cup (C\backslash B)$. **T** **F**

(iv) If N is any integer then $N - \lfloor N/3 \rfloor = \lceil 2N/3 \rceil$. **T** **F**

(v) For any $X \in \mathbb{R}$ and any $N \in \mathbb{Z}^+$, $\left\lceil \dfrac{\lceil X/N \rceil}{N} \right\rceil = \left\lceil \dfrac{X}{N^2} \right\rceil$. **T** **F**

(vi) The following functions of N are in order of increasing growth:
$$\log N\,,\ (\log N)^2\,,\ \sqrt{N}\,,\ N\log N\,,\ N^2\,.$$ **T** **F**

(vii) The running time of MERGESORT is $O(N \log N)$. **T** **F**

(viii) BUBBLESORT is a divide and conquer algorithm. **T** **F**

(ix) The GRAHAM SCAN takes $O(\log N)$ running time when given as input $N \geq 4$ points arranged in increasing polar angle. **T** **F**

(x) If $x_1 < x_2 < x_3$ then $\det \begin{bmatrix} 1 & 1 & 1 \\ x_2 & x_3 & x_1 \\ x_2^2 & x_3^2 & x_1^2 \end{bmatrix} < 0$. **T** **F**

(xi) The proposition $Q \wedge \sim Q$ is a contradiction. **T** **F**

(xii) The negation of $(\exists x)(\exists y)(\forall z)\ P(x,y,z) \Rightarrow \sim Q(x,y,z)$ is equivalent to $(\forall x)(\forall y)(\exists z)\ Q(x,y,z) \wedge \sim P(x,y,z)$. **T** **F**

(xiii) To say that a set X of integers has no largest element is to say
$$(\forall x \in X)(\exists y \in X) \quad x < y\,.$$ **T** **F**

(xiv) The recurrence $a_n = 2a_{n-1} + 3a_{n-2} + 8n - 3$ has complementary function $c_n = C_1(-1)^n + C_2(3^n)$ where C_1, C_2 are constants. **T** **F**

(xv) The recurrence $a_n = 2a_{n-1} + 3a_{n-2} + 8n - 3$ has a particular solution $p_n = -2n - 3$. **T** **F**

(xvi) If $G(z) = \dfrac{4+z}{1-2z+z^2}$ then $G(z) = \dfrac{A}{(1-z)^2} + \dfrac{B}{1-z}$ for some constants A, B. **T** **F**

(xvii) If $\displaystyle\sum_{n=0}^{\infty} a_n z^n = \frac{3-z}{1-3z+2z^2}$ then $a_3 = 36$. **T** **F**

(xviii) Given n different integers, where $n \geq 2$, there must exist two of these whose difference is divisible by $n-1$. **T** **F**

(xix) If p is prime and y is a nonzero element of $\mathbb{Z}_p$, then the function $f : \mathbb{Z}_p \to \mathbb{Z}_p$ where $f(x) = xy$ need not be a permutation. **T** **F**

(xx) If $N^5 = 5 \pmod{35}$ then $N = 0 \pmod 5$ and $N = 3 \pmod 7$. **T** **F**

[10 marks]

THIS IS THE END OF SECTION A