Section A

Write your answers to this section in the places indicated.

Place this completed sheet inside the answer booklet for Section B.

A1. Complete the following:

(i) A function \( f : A \rightarrow B \) is onto if

(ii) A function \( f : A \rightarrow B \) is one-one if

(iii) The Pigeonhole Principle says that if \( f : A \rightarrow B \) is a function where \( A \) and \( B \) are finite and \(|A| > |B|\) then

(iv) The definition of \( n(m) \) is \( \) and \( n(m) \)

counts the number of

(v) The definition of \( \binom{n}{m} \) is \( \) and \( \binom{n}{m} \)

counts the number of

(vi) The size of the power set of a finite set \( X \) is

(vii) The number of strings of eight digits using 0’s and 1’s is

(viii) The number of ways of making up bags of 5 jelly beans from a large supply of beans of 4 colours is

[10 marks]
A2. Consider each of the following statements. Circle T if you believe the statement is true. Circle F if you believe the statement is false. (Simple guessing is inadvisable. Marks may be deducted for more than three incorrect answers.)

(i) The integer 157 is a prime number. \( \text{T} \) \( \text{F} \)

(ii) If \( \lim_{N \to \infty} \frac{f(N)}{g(N)} = \infty \) and \( \lim_{N \to \infty} \frac{g(N)}{h(N)} = 0 \) then \( h(N) = O(f(N)) \). \( \text{T} \) \( \text{F} \)

(iii) If \( A, B, C \) are any sets then \( C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B) \). \( \text{T} \) \( \text{F} \)

(iv) There exist sets \( A, B \) and \( C \) such that \(|A| = 6, |B| = 3, |C| = 4, |A \cup B \cup C| = 10, |A \cap C| = 2\) and \(|A \cap B \cap C| = 1\). \( \text{T} \) \( \text{F} \)

(v) The negation of \((\forall x)(\exists y) P(x, y) \Rightarrow \sim Q(x, y)\) is equivalent to \((\exists x)(\forall y) \sim P(x, y) \land Q(x, y)\). \( \text{T} \) \( \text{F} \)

(vi) The following functions of \( N \) are in order of increasing growth: \( \sqrt{\log N}, \log \log N, \sqrt{N}, N \log N, N^{3/2} \). \( \text{T} \) \( \text{F} \)

(vii) The running time of BUBBLESORT is \( O(N \log N) \). \( \text{T} \) \( \text{F} \)

(viii) The running time of BINARYSEARCH is \( O(N \log N) \). \( \text{T} \) \( \text{F} \)

(ix) The GRAHAM SCAN takes \( O(N) \) running time when given as input \( N \geq 4 \) points arranged in increasing polar angle. \( \text{T} \) \( \text{F} \)

(x) If \( \det \begin{vmatrix} 1 & 1 & 1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} \leq 0 \) and \( P = (x_1, x_2), Q = (y_1, y_2), \) \( R = (z_1, z_2) \), then the triangle \( \triangle PQR \) is oriented anticlockwise. \( \text{T} \) \( \text{F} \)

(xi) The g.c.d. of any two Fibonacci numbers is 1. \( \text{T} \) \( \text{F} \)

(xii) The Euclidean Algorithm is a divide and conquer algorithm. \( \text{T} \) \( \text{F} \)

(xiii) MERGESORT is a divide and conquer algorithm. \( \text{T} \) \( \text{F} \)

(xiv) The recurrence \( a_n = -3a_{n-1} + 4a_{n-2} + 10 \) has complementary function \( c_n = C_1 + C_2(-4)^n \) where \( C_1, C_2 \) are constants. \( \text{T} \) \( \text{F} \)

(xv) The recurrence \( a_n = -3a_{n-1} + 4a_{n-2} + 10 \) has general solution \( a_n = C_1 + C_2(-4)^n + 2n \) where \( C_1, C_2 \) are constants. \( \text{T} \) \( \text{F} \)

(xvi) The following points are in order of increasing polar angle: \( (1, 0), (2, 2), (3, 5), (-2, 3), (-1, 2) \). \( \text{T} \) \( \text{F} \)

(xvii) The following points are vertices of some convex polygon: \( (0, 0), (-2, 3), (3, 5), (1, 0), (2, 2), (-1, 2) \). \( \text{T} \) \( \text{F} \)

(xviii) If \( \sum_{n=0}^{\infty} a_n z^n = \frac{2 + 3z}{1 - 3z + 2z^2} \) then \( a_2 = 24 \). \( \text{T} \) \( \text{F} \)

(xix) If there are 1000 people in a room then at least 3 of them are guaranteed to have the same birthday. \( \text{T} \) \( \text{F} \)

(xx) The equation \( 26x = 5 \pmod{143} \) has no integer solution. \( \text{T} \) \( \text{F} \)

[10 marks]

THIS IS THE END OF SECTION A