

## Attachment to Examination Paper (June 2005)

## MATH2969 PAPER 1

Name:

Student No.:

## Section A

Write your answers to this section in the places indicated.

Place this completed sheet inside the answer booklet for Section B.

A1. Complete the following:

- (i) A function  $f : A \rightarrow B$  is *onto* if
- (ii) A function  $f : A \rightarrow B$  is *one-one* if
- (iii) The Pigeonhole Principle says that if  $f : A \rightarrow B$  is a function where  $A$  and  $B$  are finite and  $|A| > |B|$  then
- (iv) The definition of  $n_{(m)}$  is  and  $n_{(m)}$  counts the number of
- (v) The definition of  $\binom{n}{m}$  is  and  $\binom{n}{m}$  counts the number of
- (vi) The size of the power set of a finite set  $X$  is
- (vii) The number of strings of eight digits using 0's and 1's is
- (viii) The number of ways of making up bags of 5 jelly beans from a large supply of beans of 4 colours is

[10 marks]

SEE OVER FOR QUESTION A2

**A2.** Consider each of the following statements. Circle **T** if you believe the statement is true. Circle **F** if you believe the statement is false. (Simple guessing is inadvisable. Marks may be deducted for more than three incorrect answers.)

- (i) The integer 157 is a prime number. **T F**
- (ii) If  $\lim_{N \rightarrow \infty} \frac{f(N)}{g(N)} = \infty$  and  $\lim_{N \rightarrow \infty} \frac{g(N)}{h(N)} = 0$  then  $h(N) = O(f(N))$ . **T F**
- (iii) If  $A, B, C$  are any sets then  $C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$ . **T F**
- (iv) There exist sets  $A, B$  and  $C$  such that  $|A| = 6, |B| = 3, |C| = 4, |A \cup B \cup C| = 10, |A \cap C| = 2$  and  $|A \cap B \cap C| = 1$ . **T F**
- (v) The negation of  $(\forall x)(\exists y) P(x, y) \Rightarrow \sim Q(x, y)$  is equivalent to  $(\exists x)(\forall y) \sim P(x, y) \wedge Q(x, y)$ . **T F**
- (vi) The following functions of  $N$  are in order of increasing growth:  
 $\sqrt{\log N}, \log \log N, \sqrt{N}, N \log N, N^{3/2}$ . **T F**
- (vii) The running time of BUBBLESORT is  $O(N \log N)$ . **T F**
- (viii) The running time of BINARYSEARCH is  $O(N \log N)$ . **T F**
- (ix) The GRAHAM SCAN takes  $O(N)$  running time when given as input  $N \geq 4$  points arranged in increasing polar angle. **T F**
- (x) If  $\det \begin{bmatrix} 1 & 1 & 1 \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{bmatrix} < 0$  and  $P = (x_1, x_2), Q = (y_1, y_2), R = (z_1, z_2)$ , then the triangle  $\triangle PQR$  is oriented anticlockwise. **T F**
- (xi) The g.c.d. of any two Fibonacci numbers is 1. **T F**
- (xii) The Euclidean Algorithm is a divide and conquer algorithm. **T F**
- (xiii) MERGESORT is a divide and conquer algorithm. **T F**
- (xiv) The recurrence  $a_n = -3a_{n-1} + 4a_{n-2} + 10$  has complementary function  $c_n = C_1 + C_2(-4)^n$  where  $C_1, C_2$  are constants. **T F**
- (xv) The recurrence  $a_n = -3a_{n-1} + 4a_{n-2} + 10$  has general solution  $a_n = C_1 + C_2(-4)^n + 2n$  where  $C_1, C_2$  are constants. **T F**
- (xvi) The following points are in order of increasing polar angle:  
 $(1, 0), (2, 2), (3, 5), (-2, 3), (-1, 2)$  **T F**
- (xvii) The following points are vertices of some convex polygon:  
 $(0, 0), (-2, 3), (3, 5), (1, 0), (2, 2), (-1, 2)$  **T F**
- (xviii) If  $\sum_{n=0}^{\infty} a_n z^n = \frac{2 + 3z}{1 - 3z + 2z^2}$  then  $a_2 = 24$ . **T F**
- (xix) If there are 1000 people in a room then at least 3 of them are guaranteed to have the same birthday. **T F**
- (xx) The equation  $26x = 5 \pmod{143}$  has no integer solution. **T F**

[10 marks]