

THE UNIVERSITY OF SYDNEY  
MATH2069/2969 DISCRETE MATHEMATICS

Semester 1

Quiz 2B

2006

Name (with surname underlined):

Student No. and signature:

*This quiz lasts 35 minutes and is worth 30 marks.*

1. Find the unique  $q$  and  $r$  such that  $a = qb + r$  where  $0 \leq r < |b|$  in each of the following cases:

(a)  $a = 97$ ,  $b = 10$ .

$q =$

$r =$

(b)  $a = -97$ ,  $b = -10$ .

$q =$

$r =$

(2 marks)

2. Evaluate the following determinant, including brief working:

$$\det \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ -1 & 3 & -7 \end{bmatrix}$$

Answer:

(2 marks)

(1 + 2 = 3 marks)

3. Let  $k$  be a fixed positive integer. Consider the following divide and conquer algorithm for multiplying two numbers  $X$  and  $Y$ , where  $X$  and  $Y$  each contain  $N = 2^k$  digits:

(1) Write  $X = 10^{2^{k-1}}A + B$ ,  $Y = 10^{2^{k-1}}C + D$ .

(2) Find  $AC$ ,  $BD$ ,  $(A - B)(D - C)$ .

(3) Find  $XY = 10^{2^k}AC + 10^{2^{k-1}}(AC + BD + (A - B)(D - C)) + BD$ .

(a) Which is the “divide” step?

**Answer:**

(b) Which is the “conquer” step?

**Answer:**

(c) Which is the “merge” step?

**Answer:**

(d) If we wished to apply this algorithm recursively, in which step would the algorithm call on itself?

**Answer:**

(e) Assuming the algorithm is applied recursively, write down a recurrence formula for the running time  $T(N)$  using Big-Oh notation:

**Answer:**

(1 + 1 + 1 + 1 + 1 = 5 marks)

4. Sort the following points in the plane by increasing polar angle:

$(1, 3)$ ,  $(-3, 5)$ ,  $(0, 1)$ ,  $(-1, 6)$ ,  $(2, 1)$

**Answer:**

(3 marks)

5. (a) Write out the equations when the Euclidean Algorithm is applied to find the greatest common divisor of 97 and 10.

**Answer:**

- (b) Express  $\text{g.c.d.}(97,10)$  as an integer linear combination of 97 and 10.

**Answer:**

- (c) Find  $97^{-1}$  in  $\mathbb{Z}_{10}$  and  $10^{-1}$  in  $\mathbb{Z}_{97}$ .

**Answers:**

(3 + 3 + 2 = 8 marks)

6. Today is Wednesday. What day of the week will it be after  $10^{10}$  days have elapsed? Include brief working.

**Final answer:**

(1 + 2 = 3 marks)

7. Consider each of the following statements. Circle **T** if you believe the statement is true. Circle **F** if you believe the statement is false. (If you are unsure, leave it.)

- (i) Exponential growth is faster than polynomial growth. **T F**
- (ii)  $\sum_{I=1}^N (N - I) = O(N)$ . **T F**
- (iii) The running time of BUBBLESORT is  $O(N \log N)$ . **T F**
- (iv) The running time of BINARYSEARCH is  $O(\log N)$ . **T F**
- (v) MERGESORT is a divide and conquer algorithm. **T F**
- (vi) The equation  $5x = 2 \pmod{8}$  has no integer solution. **T F**
- (vii) The equation  $2x = 5 \pmod{8}$  has some integer solution. **T F**
- (viii) No even integer  $n$  exists such that  $\text{g.c.d.}(n, n + 2) > 2$ . **T F**
- (ix) The Euclidean Algorithm is a divide and conquer algorithm. **T F**
- (x) You are given the following determinants:

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & -2 \\ -1 & 5 & 3 \end{vmatrix} = 40 \qquad \begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 0 \\ -1 & 5 & 0 \end{vmatrix} = 25$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 0 & -2 \\ -1 & 0 & 3 \end{vmatrix} = -10 \qquad \begin{vmatrix} 1 & 1 & 1 \\ 0 & 5 & -2 \\ 0 & 5 & 3 \end{vmatrix} = 25$$

Let  $P_0 = (0, 0)$ ,  $P_1 = (4, -1)$ ,  $P_2 = (5, 5)$ ,  $P_3 = (-2, 3)$ .

- (a) The triangle  $\triangle P_1 P_2 P_3$  is oriented clockwise. **T F**
- (b) The triangle  $\triangle P_0 P_3 P_2$  is oriented clockwise. **T F**
- (c) The point  $P_0$  lies inside the triangle  $\triangle P_1 P_2 P_3$ . **T F**

[ 1/2 mark for each correct answer,  
 -1/2 mark for each incorrect answer,  
 0 mark for each answer unattempted,  
 maximum total = 6 marks,  
 minimum total = -6 marks]

**END OF QUIZ**