Math 3066
Lecture 1
Mon 29/2/2016

Algebra & Logic

Lecture & tutor: David Eastwood

Lectures: Mon, Wed, Thurs 10 am
Tutorials: Mon 1 pm, Wed 9 am, Thurs 9 am

From Week 2

Distribute profile sheets to fill out.

Announcement: There will be a surprise quiz next week in the Mon, Wed or Thurs lecture.

Corollary: There is no quiz next week.

Proof: (i) If quiz is on Thursday then know after Week so no surprise. Hence no quiz on Thursday.

(ii) If quiz is on Wed then know after Mon so no surprise. Hence no quiz on Wed.

By (i) & (ii) quiz must be on Mon, so no surprise.

Hence there is no quiz next week.
Hence (*) is true if there really is a quiet next week!!
(a surprise to the lecturer as well)

Can a personal pronoun (e.g.) be believed?

Does (*) have a well-defined truth value?

Most statements are clearly true or false

e.g. $2 + 2 = 4$  true

$2 + 2 = 5$  false

What about

*I am reliable*  i.e., always tell the truth

*I am reliably unreliable*  i.e., always tell a lie?

Contemporary research:

*temporal - epistemological logics?*

incorporating time

incorporating knowledge/belief
Google: blue-eyes island problem
- blog by Terry Tao

Computer assisted proof that there is no quit.

next week:

Google: MATH 366
- Information Sheet
- Assessment Into Peer reviews

Note: no quit next week, no quitters at all. Q.E.D.

- Introductory notes on implication
- Deduction rules for Propositional/ Predicate
  Calculi
- Weekly exercises (working system).
Important theme of MATH 3066:
- exploring the "boundary" between what is (algorithmically) possible and impossible.

Birth of modern algebra related to solvability of polynomial equations:

\[
\begin{align*}
(i) & \quad ax^2 + bx + c = 0 \implies x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
(ii) & \quad ax^3 + bx^2 + cx + d = 0 \implies x = \sqrt[3]{\text{complicated formula involving } \sqrt{\frac{b^2}{9} - \frac{27ac - 3b^2}{18}} + \frac{2b}{3}} \\
(iii) & \quad ax^4 + bx^3 + cx^2 + dx + e = 0 \implies x = \sqrt[4]{\text{complicated formula involving } \sqrt[3]{\frac{27}{2}c^2 - \frac{9}{2}b^2 + \sqrt{\left(\frac{b^3}{8} - \frac{3}{4}ac + \frac{3}{16}d^2\right)^3 + \left(\frac{b^2}{2} - \frac{3}{4}ad + \frac{3}{8}c^2 + \frac{1}{8}e\right)^2}}} 
\end{align*}
\]
(16th century)
(iv) Shoch 1. Quintic polynomial equations are impossible to solve by radicals

- Abel 1824, Cadii 1830

- Birth of group theory

- Science of symmetry

- Use of field extensions, Cadii connection

Irreducible quintic equation ↔ solvable group

Unsolvability of the quintic leads to discovery of the smallest nonabelian simple group $A_5$

Involves notions of even & odd permutations, one-one & onto maps
Example of impossibility (Möbius strip)

\[ B_1 \] (bent without twist)

\[ B_2 \] (bent with twist) (Möbius strip)

**Question:** Can \( B_2 \) be deformed into \( B_1 \) continuously (without tearing)?

**Answer:** No!! (How do you know?)

**Proof:** Continuous deformations preserve the connectedness of the boundary of \( B_2 \).

(Analogous closed curve)
The boundary of \( B' \) is disconnected.

If a continuous deformation existed from \( B_2 \) to \( B' \), then the boundary of \( B' \) would be connected.  \( \times \)  (Contradiction)

Hence no such deformation exists.

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Question: Can \( B_3 \) be deformed into \( B' \), where \( B_3 \) has two "holes"?

(An argument based on connectedness of the boundary now follows.)