

# The Lower Part of the Lattice of all Varieties of Semigroups

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## 1 Abstract

A *variety* is a class of semigroups closed under the operations of taking homomorphic images, subsemigroups, and arbitrary direct products. Such a class of semigroups, according to the Birkhoff Theorem, forms an equational class. The collection of all varieties of semigroups forms a complete lattice, and that lattice is the subject of our talk. Several different approaches have been used in the literature to investigate different sublattices of this lattice, and some of these approaches are:

- (A) The lattice structure formed by the collection of all *fully invariant congruences* on free semigroups of a variety  $\mathcal{V}$  are studied, because this lattice is known to be dually isomorphic to the lattice of all subvarieties of  $\mathcal{V}$ .
- (B) Knowledge about algebraic structures and properties possessed by semigroups in a variety are useful. For example, if it is possible to express every member of a given variety as a subdirect product of semigroups from some subvarieties, then it is possible to express that variety as a join of these other subvarieties.
- (C) A direct attack on semigroup identities themselves have also been useful. Combined with some number theoretic techniques, some researchers have established all distinct sets of identities of a certain types

and have determine the hierarchy of implications and consequences. This information has led them to solve the word problem.

Our research approach has been along the lines described in (B). In this talk we will give the sketch of a certain lower part of the lattice of all semigroup varieties, explicitly describing some particular sublattices. All this progress has been possible by making use of a countably infinite family of injective endomorphisms on the lattice of all varieties of semigroups.

## References

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