Lecture 2 MATH1904

Sets

- **Membership**

  \[ x \in A \]
  
  means that \( x \) is an element of \( A \) and

  \[ x \notin A \]
  
  means that \( x \) is *not* an element of \( A \).

- **Set specification**

  Enclose the elements of the set in braces. For example

  \[ \{1, 3, 5, 7, 9\} \]
  
  Give a condition that the elements of the set must satisfy. For example

  \[ \{2x - 1 \mid 1 \leq x \leq 5\}. \]
• **The empty set**

The set with no elements at all is called the *empty set* and denoted by $\emptyset$ or $\{\}$. 

• **Subsets**

Let $A$ and $B$ be two sets. Then $B$ is said to be a *subset* of $A$ if every element of $B$ is also an element of $A$ and we write this as $B \subseteq A$. 

• **The power set**

The set of all subsets of a set $A$ is called the *power set* of $A$. 

• **Equality**

Two sets $A$ and $B$ are *equal* if they have the same elements.
• Union

Let \( A \) and \( B \) be sets. The union of \( A \) and \( B \), denoted by \( A \cup B \), is the set of elements that are in either \( A \) or \( B \). That is,

\[
A \cup B = \{ x \mid x \in A \text{ or } x \in B \}.
\]

• Intersection

Let \( A \) and \( B \) be sets. The intersection of \( A \) and \( B \), denoted by \( A \cap B \), is the set of elements that are in both \( A \) and \( B \). That is,

\[
A \cap B = \{ x \mid x \in A \text{ and } x \in B \}.
\]

• Complement

Let \( A \) and \( B \) be sets. The set \( A \setminus B \), called the complement of \( B \) in \( A \), is defined to be the set of elements that are in \( A \) but not in \( B \). That is,

\[
A \setminus B = \{ x \mid x \in A \text{ and } x \notin B \}.
\]
• **Cardinality**

The number of different elements in the set $A$ is called the *cardinality* (or *size*) of $A$ and written $|A|$.

**The algebra of set theory**

Laws similar to ordinary algebra hold for the operations of union and intersection of sets but because sets are different from numbers there are other laws which have no counterpart in the algebra of numbers. A sample of the laws of set theory is given by the following list of identities which hold for all sets $A$, $B$, $C$ and $X$. 
1. \( A \cup A = A, \ A \cap A = A \)

2. \( A \cup \emptyset = A, \ A \cap \emptyset = \emptyset \)

3. \( A \cup B = B \cup A, \ A \cap B = B \cap A \)

4. \( A \cup (B \cup C) = (A \cup B) \cup C, \)
   \[ A \cap (B \cap C) = (A \cap B) \cap C \]

5. \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C), \)
   \[ A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \]

6. \( A \cup B \subseteq X \) if and only if \( A \subseteq X \) and \( B \subseteq X \)

7. \( X \subseteq A \cap B \) if and only if \( X \subseteq A \) and \( X \subseteq B \).
De Morgan’s laws

If $A$ and $B$ are subsets of a set $X$, then

- $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$

- $X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$.  

- If $A$ is a subset of the set $X$, then
  
  $X \setminus (X \setminus A) = A.$