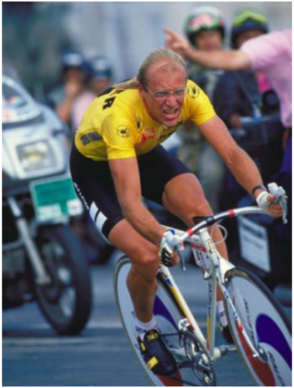
Bicycle Aerodynamics, or:

How to lose the Tour de France.

Duncan Sutherland

ΣUMS

Background



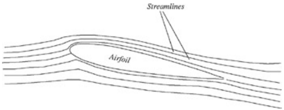
1989 Tour de France final stage. Greg LeMond (right) beat Laurent Fignon (left) by 8 seconds. The closest margin in Tour de France history.

From Wikipedia: " Fignon, who rode after LeMond, lost 58 seconds during the stage, and although he became third in the stage, he lost the lead to LeMond. It was calculated afterwards that if Fignon had cut off his ponytail, he would have reduced his drag that much that he would have won the Tour.[McCann]"

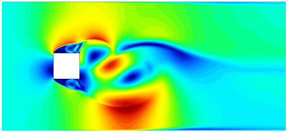
Jolly good, but how is such a problem approached? Paper by Thijis Defraeye, et al. "Aerodynamic study of different cyclist positions: CFD analysis and full-scale wind-tunnel tests." Journal of Biomechanics, v43, i7, pp 1262-1286 (2010).

Streamlined and bluff bodies

- Streamlined bodies: flow follows the contours of the body. Example: airfoils.
- Bluff bodies: flow around the body separates and vorticies are formed by rolling up of shear layers. Example: A brick.
- Cyclists are combinations of these types of body. Areas such as the helmet may be quite streamlined, but areas such as the torso may be bluff bodies.



(a) Airfoil



(b) Brick

Resistance

- A cyclist's performance is affected by the resistance, eg, drag, friction and gradients
- At the highest speeds 50kms/h and above, most of the resistance is aerodynamic drag, mostly accounted for by the position of the cyclist's body.
- Two sources of drag: form drag contributes more than 90% of the drag and viscous drag, caused by roughness on the surface of the cyclist.

Dimensional analysis and the drag equation

Dimensional analysis is way of deriving an equation for an unknown quantity without thinking too hard:

- Take variables of interest: drag F_D, velocity u, density ρ, area L^2 and viscosity ν
- Write
$$f_a(F_D, u, \rho, L^2, \nu) = 0$$
and note that 0 is dimensionless, thus the function must be dimensionless.
- Rearranging into dimensionless groups
$$f_b\left(\frac{F_D}{\rho L^2 u^2}, \frac{uL}{\nu}\right) = 0$$
since only F_D is unknown:
- $F_D = \frac{1}{2}\rho L^2 u^2 f_c(Re)$, $Re = \frac{uL}{\nu}$ and then define $C_D = f_c(Re)$. $C_D \approx 0.23$.

Fluid Dynamics

Fluid mechanics is basically concerned with solving the Navier-Stokes equation and the continuity equation

$$\rho\left(\frac{\partial u_i}{\partial t}+u_j\frac{\partial u_i}{\partial x_j}\right)=-\frac{\partial p}{\partial x_i}+\frac{\partial T_{ij}}{\partial x_i}$$
$$\frac{\partial \rho}{\partial t}+u_j\frac{\partial \rho}{\partial x_i}+\rho\frac{\partial u_j}{\partial x_j}=0$$

Simplifying assumptions: incompressibility, $\frac{\partial u_i}{\partial x_i}=0$; Newtonian fluid, $T_{ij}=\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ These are complicated and we want to use a computer. Problems are cost, stability and accuracy.

Reynolds Averaged Navier Stokes

For an incompressible, viscous, Newtonian fluid, we may decompose velocity, u_i into the time averaged part U_i and the fluctuating part, with zero mean, $\tilde{u}$:

$$u_i = U_i + \tilde{u}_i$$
$$p_i = P_i + \tilde{p}_i$$
$$T_{ij} = \bar{T}_{ij} + \tau_{ij}$$

This gives the RANS equation:

$$\rho\left(\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j}\right) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}(\bar{T}_{ij} - \rho\langle \tilde{u}_i \tilde{u}_j \rangle)$$

Turbulence Closure Problem: use models of turbulence to determine $\langle \tilde{u}_i \tilde{u}_j \rangle$.

Free pizza

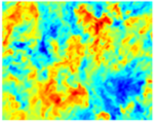
Large Eddy Simulation

Low-pass filter applied to the Navier-Stokes equation.

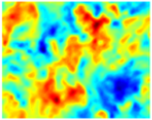
$$\varphi(\bar{\mathbf{x}}, t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \varphi(\mathbf{r}, t')G(\mathbf{r}-\mathbf{x}, t-t')dt'd\mathbf{r}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{u}_i\bar{u}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + 2\nu\frac{\partial}{\partial x_j}\bar{S}_{ij} - \frac{\partial \tau^r_{ij}}{\partial x_j}$$

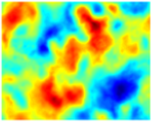
S_{ij} is the rate of strain tensor. The residual stress tensor $\tau^r_{ij} = L_{ij} + C_{ij} + R_{ij}$ L_{ij} represents interation of large scales, C_{ij} cross scale interations and R_{ij} is subscale stress.



(c) Direct simulation



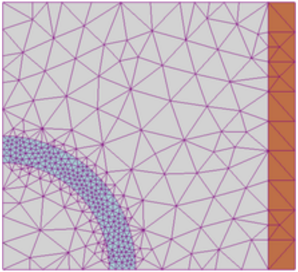
(d) LES simulation $\Delta = L/32$



(e) LES simulation $\Delta = L/16$

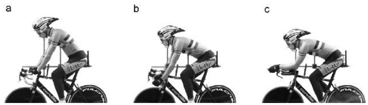
Finite Element Method

FEM is a means of approximating the solution to a PDE on a mesh, using certain basis functions eg: peicewise polynomials. The mesh can cope with complicated geometries and the computation is relatively cheap.



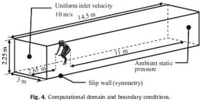
Defraeye paper

- Comparison of the area drag L^2C_D, of different cyclist positions.
- Wind tunnel experiments and a comparison of LES and RANS calculations based upon the cyclist's position alone.
- Experimental setup of a bicycle was photographed, digitised and the cyclist was smoothed into a mesh suitable for finite element computations.

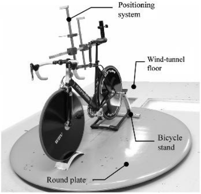


Meshing

The mesh is generated by a 3D laser scanning and digitising system. The bicycle, stand and other pieces of experimental equipment were omitted from the computational domain.



(f) Computational domain



(g) Experimental setup

Comparison of simulation and experiment

Drag area of cyclist and bicycle ($AC_{D,\text{cyclist and bicycle}}$), drag area of cyclist ($AC_{D,\text{cyclist}}$) and ratio of these drag areas for different positions for the wind-tunnel experiments.

Position	$AC_{D,\text{cyclist and bicycle}}$ (m^2)	$AC_{D,\text{cyclist}}$ (m^2)	$\frac{AC_{D,\text{cyclist}}}{AC_{D,\text{cyclist and bicycle}}}$ (%)
UP	0.270	0.193	72
DP	0.243	0.167	68
TTP	0.211	0.134	64

Drag area of cyclist from CFD simulations ($AC_{D,\text{cyclist, CFD}}$) and differences with wind-tunnel experiments ($AC_{D,\text{cyclist}}$), i.e. without bicycle setup (see Table 2), for different positions for RANS and LES.

Position	Turbulence modelling	$AC_{D,\text{cyclist, CFD}}$ (m^2)	$\frac{AC_{D,\text{cyclist,CFD}} - AC_{D,\text{cyclist}}}{AC_{D,\text{cyclist}}}$ (%)
UP	RANS	0.219	13
	LES	0.219	13
DP	RANS	0.179	7
	LES	0.172	3
TTP	RANS	0.150	12
	LES	0.142	6

Conclusion

Maurice Garin, disqualified in 1904 for catching a train.