MATHS WORKSHOPS
Functions

Business School
Outline

Overview of Functions

Quadratic Functions

Exponential and Logarithmic Functions

Summary and Conclusion
Outline

Overview of Functions

Quadratic Functions

Exponential and Logarithmic Functions

Summary and Conclusion
A function, $f$, is a *mapping* from one value, $X$, to another value, $Z$:

$$f : X \mapsto Z.$$ 

Think of the function, $f$, as a *machine* that takes an input, $X$, then transforms it in some way and outputs the result: $Z$.

**Example** ($f(x) = 2x - 3$)

The function $f(x) = 2x - 3$ has the following mapping:

- $f(3) = 2 \times 3 - 3 = 3$ so $f(x)$ maps 3 to 3.
- $f(2) = 2 \times 2 - 3 = 1$ so $f(x)$ maps 2 to 1.
- $f(1) = 2 \times 1 - 3 = -1$ so $f(x)$ maps 1 to $-1$.
- $f(0) = 2 \times 0 - 3 = -3$ so $f(x)$ maps 0 to $-3$. 
Functions

Definition (Function)

A function, \( f \), is a \textit{mapping} from one value, \( X \), to another value, \( Z \):

\[ f : X \mapsto Z. \]

Think of the function, \( f \), as a \textit{machine} that takes an input, \( X \), then transforms it in some way and outputs the result: \( Z \).

Example (Where have we seen functions before?)

We've been working with functions already:

- Linear Functions: \( f(x) = ax + b \)
- Quadratic Functions: \( f(x) = ax^2 + bx + c \) (today's lesson)

- Sometimes functions are written as \( y = f(x) \). For example you may see, \( y = ax + b \) instead of \( f(x) = ax + b \).
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Summary and Conclusion
## Quadratic Function

### Definition (Quadratic Function)

A **quadratic function** takes the form:

\[ f(x) = ax^2 + bx + c \]

where \( a, b \) and \( c \) are parameters and \( x \) is a variable.

### Key point

The key point is that there is a term in the function involving the **square** of the variable, \( x^2 \).

### Definition (Parabola)

The graph of a quadratic functions is a curve often referred to as a **parabola**.
Factorising quadratic function

Definition (Factorisation)

Quadratic functions can be factorised to take the form:

\[ f(x) = (x + d)(x + e) \]

(factorised)

\[ = (x + d)x + (x + d)e \]

\[ = x^2 + dx + xe + de \]

\[ = x^2 + (d + e)x + de \] (expanded)

Example (Expand the following factorised quadratic)

\[(x - 2)(x + 1) = \]
Factorising quadratic function

Definition (Factorisation)

Quadratic functions can be factorised to take the form:

\[ f(x) = (x + d)(x + e) \]  \hspace{1cm} (factorised)
\[ = (x + d)x + (x + d)e \]
\[ = x^2 + dx + xe + de \]
\[ = x^2 + (d + e)x + de \]  \hspace{1cm} (expanded)

Example (Expand the following factorised quadratic)

\[ (x - 2)(x + 1) = (x - 2)x + (x - 2) \times 1 \]
Factorising quadratic function

**Definition (Factorisation)**

Quadratic functions can be factorised to take the form:

\[ f(x) = (x + d)(x + e) \quad \text{(factorised)} \]

\[ = (x + d)x + (x + d)e \]

\[ = x^2 + dx + xe + de \]

\[ = x^2 + (d + e)x + de \quad \text{(expanded)} \]

**Example (Expand the following factorised quadratic)**

\[(x - 2)(x + 1) = (x - 2)x + (x - 2) \times 1\]

\[= x^2 - 2x + x - 2\]

\[= x^2 - x - 2\]
Factorising quadratic functions

Example (Factorise \( f(x) = x^2 + 12x + 32 \))

We factorise this to \((x + d)(x + e)\) by matching the expansion:

\[
x^2 + (d + e)x + de
\]

with our example:

\[
x^2 + 12x + 32.
\]

I.e. we try to find factors \( d \) and \( e \) such that:

- Their product is \( de = 32 \) so that the constants match.
- Their sum is \( d + e = 12 \) so the coefficients of \( x \) match.
- The factors are the two numbers whose sum is 12 and their product is 32. By trial and error we notice that \( 4 + 8 = 12 \) and \( 4 \times 8 = 32 \):

\[
f(x) = (x + 4)(x + 8).
\]
Factorising a more complex quadratic

The general form for a quadratic function is:

\[ f(x) = ax^2 + bx + c \]

Example (Factorise \( f(x) = 2x^2 + 3x - 5 \))

Here we want to break up the middle term to help factorisation. To do this we find two numbers whose:

- **P** product is \( ac = 2 \times -5 = -10 \)
- **S** sum is \( b = 3 \)
- **F** Using **trial and error** we find suitable candidates \(-2\) and \(5\).

We can then re-write the original equation as:

\[
2x^2 + 3x - 5 = 2x^2 - 2x + 5x - 5 \\
= 2x(x - 1) + 5(x - 1) \\
= (2x + 5)(x - 1)
\]
Your turn to factorise.

Factorise this expression

\[ x^2 + 5x + 6 \]

in the form \((x + d)(x + e)\).

\[ \text{P Product:} \]

\[ \text{S Sum:} \]

\[ \text{F Factors:} \]
Your turn to factorise... 

Factorise this expression

\[x^2 + 5x + 6\]

in the form \((x + d)(x + e)\).

P  Product: Need two numbers that multiply to give 6
S  Sum: Need two numbers that sum to give 5
F  Factors:
Your turn to factorise. . .

Factorise this expression

\[ x^2 + 5x + 6 \]

in the form \((x + d)(x + e)\).

**P** Product: Need two numbers that multiply to give 6

**S** Sum: Need two numbers that sum to give 5

**F** Factors: \(2 + 3 = 5\) and \(2 \times 3 = 6\)

Therefore the factorisation is:

\[(x + 2)(x + 3)\]

You can verify this by expanding it out again!
Your turn to factorise. . .

Factorise this expression

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in the form \((x + d)(x + e)\).

- **P** Product: Need two numbers that multiply to give 6
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- **F** Factors: \(2 + 3 = 5\) and \(2 \times 3 = 6\)

Therefore the factorisation is:

\[(x + 2)(x + 3)\]

- You can verify this by expanding it out again!
Factorising using the cross method

In general, we can factorise \( ex^2 + fx + g \) using

\[
ac x^2 + (ad + bc)x + bd = (ax + b)(cx + d)
\]

by finding \( a, b, c, d \) such that \( ac = e, bd = g \) and \( ad + bc = f \), using the cross method:

\[
\begin{align*}
ax & \quad b \\
\downarrow & \quad \downarrow \\
\downarrow & \quad \downarrow \\
cx & \quad d
\end{align*}\]

check if \( ad + cb = f \)

1. Pick an \( a \) and a \( c \) such that \( ac = e \)
2. Pick a \( b \) and a \( d \) such that \( bd = g \)
3. If \( ad + cb = f \) then you have the solution.
   If \( ad + cb \neq f \) go back to Step 1.
Your turn to factorise with a harder example. . .

$$2x^2 + 11x + 5$$

Method 1: Using a PSF-type approach:

**P** Product:

**S** Sum:

**F** Factors:
Your turn to factorise with a harder example... 

\[2x^2 + 11x + 5\]

Method 1: Using a PSF-type approach:

P Product: Need two numbers that multiply to give \(2 \times 5 = 10\)

S Sum: Need two numbers that sum to give 11

F Factors:
Your turn to factorise with a harder example. . .

\[2x^2 + 11x + 5\]

Method 1: Using a PSF-type approach:

P Product: Need two numbers that multiply to give \(2 \times 5 = 10\)

S Sum: Need two numbers that sum to give 11

F Factors: \(10 + 1 = 11\) and \(10 \times 1 = 10\)
Your turn to factorise with a harder example.

\[2x^2 + 11x + 5\]

Method 1: Using a PSF-type approach:

- **P** Product: Need two numbers that multiply to give \(2 \times 5 = 10\)
- **S** Sum: Need two numbers that sum to give 11
- **F** Factors: \(10 + 1 = 11\) and \(10 \times 1 = 10\)

We can use this to re-write the original expression:

\[
2x^2 + 11x + 5 = 2x^2 + x + 10x + 5 \\
= x(2x + 1) + 5(2x + 1) \\
= (x + 5)(2x + 1).
\]
Your turn to factorise with a harder example... 

\[ 2x^2 + 11x + 5 \]

Method 2: the cross method to factorise this as:

\[ acx^2 + (ad + bc)x + bd = (ax + b)(cx + d) \]

Here we want to find \( a, b, c, d \) such that \( ac = 2, bd = 5 \) and \( ad + bc = 11 \).

\[
\begin{align*}
ax &= x & b &= 5 \\
\times & & \times \\
& & & \\
\times & & \times \\
& & & \\
cx &= 2x & d &= 1
\end{align*}
\]

1. Pick \( a = 1 \) and \( c = 2 \) such that \( ac = 2 \)
2. Pick \( b = 5 \) and \( d = 1 \) such that \( bd = 5 \)
3. Check if \( ad + cb = 11 \). Here \( 1 \times 1 + 5 \times 2 = 11 \), so we have a solution: \( (x + 5)(2x + 1) \)
Graphing quadratic functions

One way to graph quadratic functions would be to plot some points and join them. Consider the function, \( f(x) = x^2 \):

<table>
<thead>
<tr>
<th>( x )</th>
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![Graph of quadratic function](attachment:image.png)
Graphing quadratic functions

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Graphing quadratic functions

Consider quadratic functions of the form $y = ax^2$.

- What does $a$ do?
Graphing quadratic functions

Consider quadratic functions of the form \( y = ax^2 \).

- What does \( a \) do?

\[
\begin{align*}
f(x) & \quad 3x^2 \\
x^2 &
\end{align*}
\]
Graphing quadratic functions

Consider quadratic functions of the form $y = ax^2$.

- What does $a$ do?

- $a$ changes the “steepness” of the curve.
Graphing quadratic functions

Consider quadratic functions of the form $y = ax^2$.

- The sign of $a$ determines whether the parabola is convex (smile) or concave (frown).
Graphing quadratic functions

Consider quadratic functions of the form $f(x) = x^2 + c$.

- What does $c$ do?
Graphing quadratic functions

Consider quadratic functions of the form \( f(x) = x^2 + c \).

- What does \( c \) do?
Graphing quadratic functions

Consider quadratic functions of the form $f(x) = x^2 + c$.

- What does $c$ do?
- $c$ moves the curve up and down.
Graphing quadratic functions

Consider quadratic functions of the form \( f(x) = ax^2 + bx + c \)

- What does \( b \) do?
Graphing quadratic functions

Consider quadratic functions of the form $f(x) = ax^2 + bx + c$

- What does $b$ do?
Graphing quadratic functions

Consider quadratic functions of the form $f(x) = ax^2 + bx + c$

- What does $b$ do?

- $b$ moves the curve from side to side
Finding the roots graphically

Definition (Roots)

The point(s) at which the quadratic function crosses the $x$ axis are called the roots of the function.

Example (Finding the roots of $f(x) = x^2 + 3x$ graphically)

![Graph showing the roots of $f(x) = x^2 + 3x$.]
Finding the roots graphically

**Definition (Roots)**

The point(s) at which the quadratic function crosses the $x$ axis are called the **roots** of the function.

**Example (Finding the roots of $f(x) = x^2 + 3x$ graphically)**

The roots occur at $x = -3$ and $x = 0$. 
Finding the roots algebraically

- The definition says that the roots are “The point(s) at which the quadratic function crosses the $x$ axis.”
- Mathematically this is when $f(x) = 0$.
- This is easiest to find using the factorised form of $f(x)$.

Example (Finding the roots of $f(x) = x^2 + 3x$)

1. Factorise $f(x)$:
   
   $$f(x) = x^2 + 3x$$
   
   $$= x(x + 3).$$

2. Work out the values of $x$ for which $f(x) = 0$ is true.
   - When $x = 0$ then $x(x + 3) = 0$.
   - When $x = -3$ then $x(x + 3) = 0$.

Therefore the roots are $x = 0$ and $x = -3$. 
What if there aren’t any roots?

The function $x^2 + 2x + 1$ doesn’t cross the $x$ axis at all!

Definition (Discriminant)

The function $f(x) = ax^2 + bx + c$ will only have root(s) if

$$b^2 - 4ac \geq 0.$$ 

• $b^2 - 4ac$ is known as the discriminant.
The Quadratic Formula

**Definition (Quadratic Formula)**

If the quadratic function \( f(x) = ax^2 + bx + c \) has roots, they can always be found using the quadratic formula:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.
\]

- Note that the square root of the discriminant is in the quadratic formula.
- This result suggests why there are no real roots unless \( b^2 - 4ac \geq 0 \). You cannot take the square root of a negative number.\(^1\)

\(^1\)You actually can but the solution is an imaginary number!
The Quadratic Formula

Example (Finding the roots of \( f(x) = x^2 + 3x \))

Here \( a = 1 \), \( b = 3 \) and \( c = 0 \) so

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times 0}}{2 \times 1}
\]

\[
x = \frac{-3 \pm \sqrt{9 - 0}}{2}
\]

\[
x = \frac{-3 \pm 3}{2}
\]

\[
x = \frac{-3 + 3}{2} \quad \text{AND} \quad \frac{-3 - 3}{2}
\]

\[
x = 0 \quad \text{AND} \quad -3.
\]
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Other common functions

Example (Exponential function)

\[ f(x) = a e^{cx+b} \]

where \( a, b, c \) and \( e \) are constants

- The graph of \( f(x) = e^x \) is upward-sloping, and increases faster as \( x \) increases.
- The graph is always above the \( x \)-axis but gets arbitrarily close to it for negative \( x \): the \( x \)-axis is an asymptote.
Other common functions

Example (Logarithmic function)

\[ f(x) = a \log(cx + b) \]

where \( a, b \) and \( c \) are constants.

- The graph of \( f(x) = \log(x) \) slowly grows to positive infinity as \( x \) increases.
- The graph is always to the right of the \( y \)-axis but gets arbitrarily close to it for small \( x \): the \( y \)-axis is an asymptote.
Relationship between logs and exponentials

- In $y = \log_a(b)$, $a$ is known as the **base** of the log.
- We can change the base of the log using the relationship:

  \[
  \log_a(b) = \frac{\log_c(b)}{\log_c(a)}.
  \]

- Using this relationship, it is clear that:

  \[
  \log_a(a) = \frac{\log_c(a)}{\log_c(a)} = 1.
  \]

- If $y = \log_a(x)$ then $x = a^y$
- Equivalently, if $y = a^x$ then $x = \log_a y$.
- If the **base** is the same, then the log function is the inverse of the exponential function:

  \[
  a^{\log_a(x)} = x \quad \text{just like} \quad \frac{ax}{a} = x.
  \]
Natural Logs

- We typically work with log base $e \approx 2.7182818 \ldots$
- $\log_e(x)$ is often written as $\ln(x)$.
- If the base is left off the log, it’s assumed $\log(x) = \log_e(x)$ (unless it’s on your calculator, in which case it means $\log_{10}$).
- Note that the usual relations hold:
  
  $$y = e^x$$
  $$\ln(y) = \ln(e^x)$$ (taking $\log_e$ of both sides)
  $$\ln(y) = x$$

- Also,
  
  $$y = \ln(x)$$
  $$e^y = e^{\ln(x)}$$ (exponentiating both sides)
  $$e^y = x$$
Exponential and Log rules

Exponentiation:

- \( a^0 = 1 \) for all \( a \neq 0 \).
- \( a^{-1} = \frac{1}{a} \)
- \( a^x a^y = a^{x+y} \)

Logarithms:

- \( \log(x^y) = y \log(x) \)
- \( \log(xy) = \log(x) + \log(y) \)
- \( \log \left( \frac{x}{y} \right) = \log(xy^{-1}) = \log(x) + \log(y^{-1}) = \log(x) - \log(y) \)
- \( \log(1) = 0 \)

\( \frac{a^x}{a^y} = a^{x-y} \)
- \( (a^x)^y = a^{xy} \)
- \( (ab)^x = a^x b^x \)
Your Turn . . .

Solve the following equations for $x$

1. $\ln(x) = 2 \implies x =$
2. $\log_2 \frac{y}{3} = 4 \implies y =$

Simplify the following expressions

1. $e^{\ln 5} =$
2. $\ln \sqrt{e} =$
3. $e^{x + \ln x} =$
4. $\ln(1 + x) - \ln(1 - x) =$
5. $\frac{\ln(1 + x)}{\ln(e^2)} =$
6. $\log_3 3^q =$
Your Turn . . .

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Your Turn . . .

Solve the following equations for $x$

1. $\ln(x) = 2 \implies x = e^2$
2. $\log_2 \frac{y}{3} = 4 \implies y = 3 \times 2^4 = 48$

Simplify the following expressions

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Your Turn . . .

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1. $\ln(x) = 2 \implies x = e^2$
2. $\log_2 \frac{y}{3} = 4 \implies y = 3 \times 2^4 = 48$

Simplify the following expressions

1. $e^{\ln 5} = 5$
2. $\ln \sqrt{e} = $
3. $e^{x + \ln x} = $
4. $\ln(1 + x) - \ln(1 - x) = $
5. $\frac{\ln(1 + x)}{\ln(e^2)} = $
6. $\log_3 3^q = $
Your Turn . . .

Solve the following equations for $x$

1. $\ln(x) = 2 \implies x = e^2$
2. $\log_2 \frac{y}{3} = 4 \implies y = 3 \times 2^4 = 48$

Simplify the following expressions

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2. $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2}$
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4. $\ln(1 + x) - \ln(1 - x) =$
5. $\frac{\ln(1 + x)}{\ln(e^2)} =$
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Your Turn . . .

Solve the following equations for \( x \)

1. \( \ln(x) = 2 \implies x = e^2 \)
2. \( \log_2 \frac{y}{3} = 4 \implies y = 3 \times 2^4 = 48 \)

Simplify the following expressions

1. \( e^{\ln 5} = 5 \)
2. \( \ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2} \)
3. \( e^x + \ln x = e^x e^{\ln x} = e^x x \)
4. \( \ln(1 + x) - \ln(1 - x) = \ln \left( \frac{1 + x}{1 - x} \right) \)
5. \( \frac{\ln(1 + x)}{\ln(e^2)} = \)
6. \( \log_3 3^q = \)
Your Turn . . .

Solve the following equations for $x$

1. $\ln(x) = 2 \implies x = e^2$
2. $\log_2 \frac{y}{3} = 4 \implies y = 3 \times 2^4 = 48$

Simplify the following expressions

1. $e^{\ln 5} = 5$
2. $\ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2}$
3. $e^{x+\ln x} = e^x e^{\ln x} = e^x x$
4. $\ln(1 + x) - \ln(1 - x) = \ln \left( \frac{1 + x}{1 - x} \right)$
5. $\frac{\ln(1 + x)}{\ln(e^2)} = \frac{\ln(1 + x)}{2 \ln(e)} = \frac{1}{2} \ln(1 + x)$
6. $\log_3 3^q =$
Your Turn . . .

Solve the following equations for \( x \)

1. \( \ln(x) = 2 \quad \Rightarrow \quad x = e^2 \)
2. \( \log_2 \frac{y}{3} = 4 \quad \Rightarrow \quad y = 3 \times 2^4 = 48 \)

Simplify the following expressions

1. \( e^{\ln 5} = 5 \)
2. \( \ln \sqrt{e} = \ln e^{1/2} = \frac{1}{2} \ln e = \frac{1}{2} \)
3. \( e^{x+\ln x} = e^x e^{\ln x} = e^x x \)
4. \( \ln(1 + x) - \ln(1 - x) = \ln \left( \frac{1 + x}{1 - x} \right) \)
5. \( \frac{\ln(1 + x)}{\ln(e^2)} = \frac{\ln(1 + x)}{2 \ln(e)} = \frac{1}{2} \ln(1 + x) \)
6. \( \log_3 3^q = q \log_3 3 = q \)
Solve the following equations for $x$

1. \[ \ln(2x + 1) = \ln(10 - x) \]

2. \[ 2^{3x+1} = 4^x \]

3. \[ \ln(2x + 3) = 3 \]

4. \[ 5^{x+1} = 200 \]
Solve the following equations for $x$

1. \[ \ln(2x + 1) = \ln(10 - x) \]
   \[ \exp\{\ln(2x + 1)\} = \exp\{\ln(10 - x)\} \]
   \[ 2x + 1 = 10 - x \]
   \[ x = 3 \]

2. \[ 2^{3x+1} = 4^x \]

3. \[ \ln(2x + 3) = 3 \]
   \[ \exp\{\ln(2x + 3)\} = \exp(3) \]
   \[ 2x + 3 = e^3 \]

4. \[ 5^{x+1} = 200 \]
Solve the following equations for \( x \)

1. \[ \ln(2x + 1) = \ln(10 - x) \]
   \[ \exp\{\ln(2x + 1)\} = \exp\{\ln(10 - x)\} \]
   \[ 2x + 1 = 10 - x \]
   \[ x = 3 \]

2. \[ 2^{3x+1} = 4^x \]
   \[ 2^{3x+1} = 2^{2x} \]
   \[ \log_2(2^{3x+1}) = \log_2(2^{2x}) \]
   \[ 3x + 1 = 2x \]
   \[ x = -1 \]

3. \[ \ln(2x + 3) = 3 \]

4. \[ 5^{x+1} = 200 \]
Solve the following equations for $x$

1. $\ln(2x + 1) = \ln(10 - x)$
   
   $\exp\{\ln(2x + 1)\} = \exp\{\ln(10 - x)\}$
   
   $2x + 1 = 10 - x$
   
   $x = 3$

2. $2^{3x+1} = 4^x$

   $2^{3x+1} = 2^{2x}$

   $\log_2(2^{3x+1}) = \log_2(2^{2x})$

   $3x + 1 = 2x$

   $x = -1$

3. $\ln(2x + 3) = 3$

   $\exp\{\ln(2x + 3)\} = e^3$

   $2x + 3 = e^3$

   $x = \frac{e^3 - 3}{2}$

4. $5^{x+1} = 200$
Solve the following equations for \( x \)

1. \[
\ln(2x + 1) = \ln(10 - x)
\]
\[
\exp\{\ln(2x + 1)\} = \exp\{\ln(10 - x)\}
\]
\[
2x + 1 = 10 - x
\]
\[
x = 3
\]

2. \[
2^{3x+1} = 4^x
\]
\[
2^{3x+1} = 2^{2x}
\]
\[
\log_2(2^{3x+1}) = \log_2(2^{2x})
\]
\[
3x + 1 = 2x
\]
\[
x = -1
\]

3. \[
\ln(2x + 3) = 3
\]
\[
\exp\{\ln(2x + 3)\} = e^3
\]
\[
2x + 3 = e^3
\]
\[
x = \frac{e^3 - 3}{2}
\]

4. \[
5^{x+1} = 200
\]
\[
\ln(5^{x+1}) = \ln(200)
\]
\[
(x + 1) \ln(5) = \ln(200)
\]
\[
x + 1 = \frac{\ln(200)}{\ln(5)}
\]
\[
x = \frac{\ln(200)}{\ln(5)} - 1
\]
Outline

Overview of Functions

Quadratic Functions

Exponential and Logarithmic Functions

Summary and Conclusion
Summary

- parameters and variables
- substitution and solving equations
- dependent vs independent variable
- linear function: \( f(x) = y = ax + b \)
- slope (is it a parameter or a variable?)
- intercept (is it a parameter or a variable?)
- finding the equation for a linear function given a plot
- plotting a linear function given an equation
- recognising quadratic functions
- factorising and expanding quadratics
- finding the roots of a quadratic using \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)
- graphs of quadratic equations (parabolas)
Coming up...

<table>
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<tr>
<th>Week 5: Simultaneous Equations and Inequalities</th>
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<tbody>
<tr>
<td>• Algebraic and graphical solutions to simultaneous equations</td>
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<td>• Understanding and solving inequalities</td>
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<th>Week 6: Differentiation</th>
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<tbody>
<tr>
<td>• Theory and rules of Differentiation</td>
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<tr>
<td>• Differentiating various functions and application of Differentiation</td>
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Additional Resources

- Test your knowledge at the University of Sydney Business School MathQuiz:

- Additional resources on the Maths in Business website
  sydney.edu.au/business/learning/students/maths

- The University of Sydney Mathematics Learning Centre has a number of additional resources:
  - Basic concepts in probability notes
  - Sigma notation notes
  - Permutations and combinations notes

- There’s also tonnes of theory, worked questions and additional practice questions online. All you need to do is Google the topic you need more practice with!
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• Questions, comments, feedback? Let us know at business.maths@sydney.edu.au