MSH4

Fundamentals of Statistical Consulting

Week 10 **Comparison of two time series**

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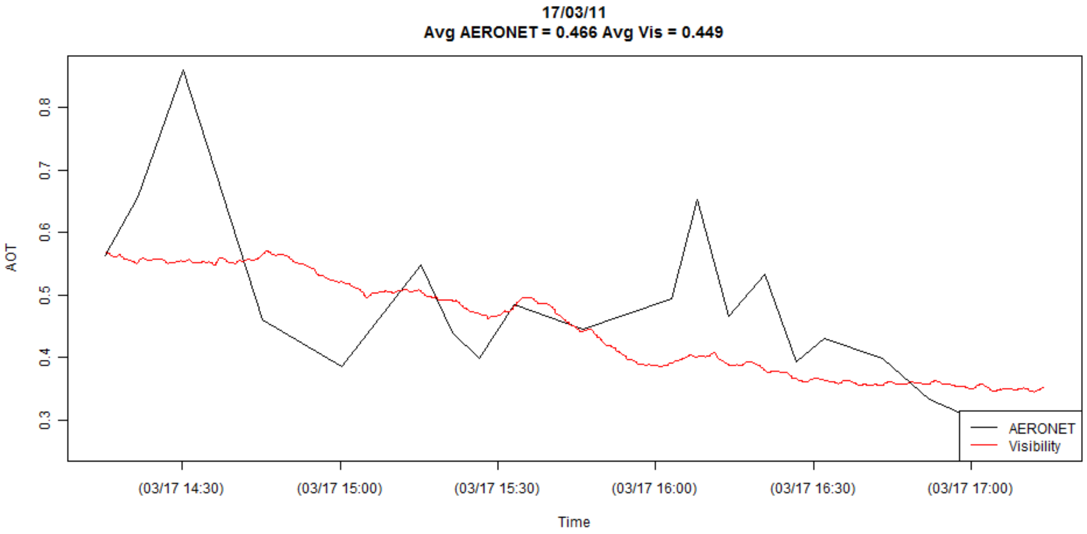
Important points

1. Difference in sampling frequencies.

   Erase the connecting lines and display only the points corresponding to actual data.



2. Difference in length.
3. What to compare?

   Process or model (time series generator)?

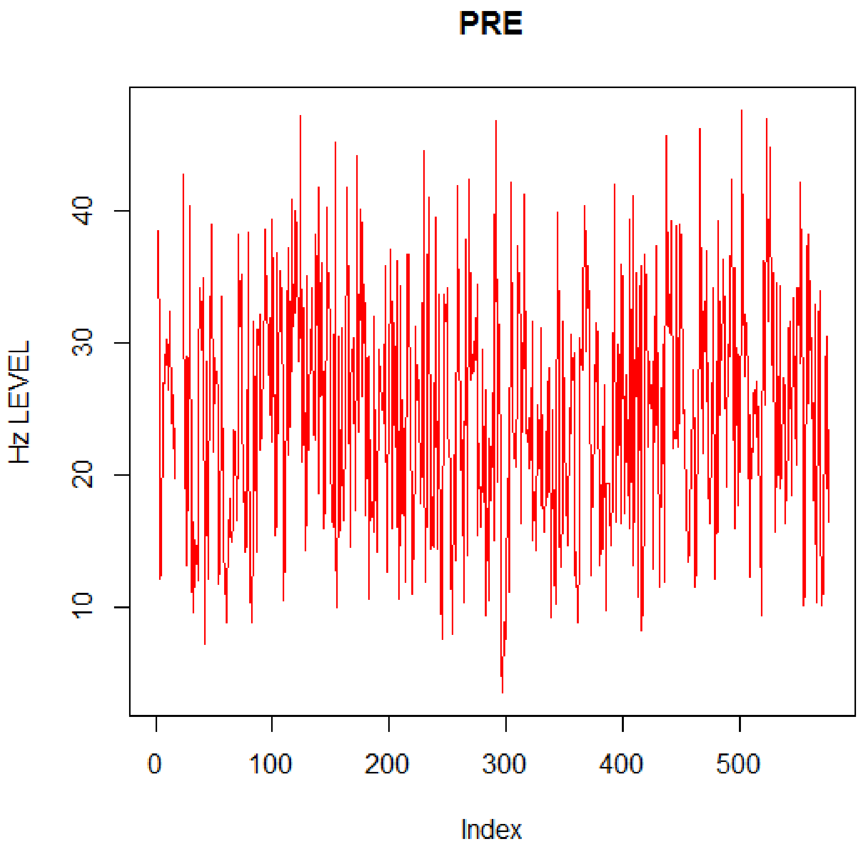
   Values, trends, patterns or periodicity?

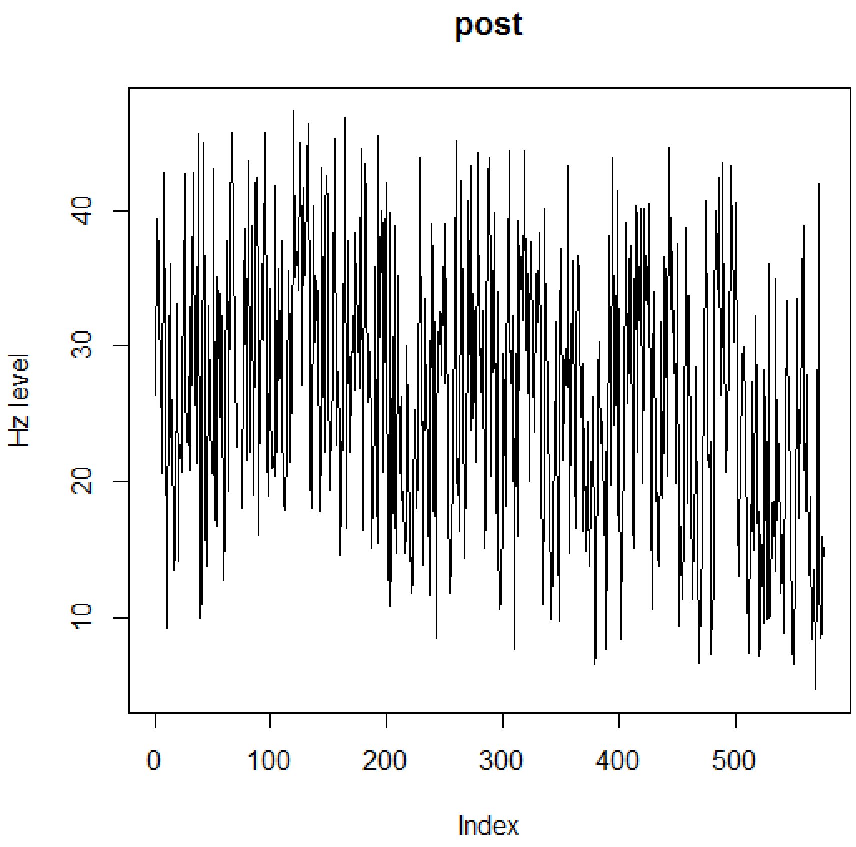
   Observed or forecast?

Some basic graphical comparison

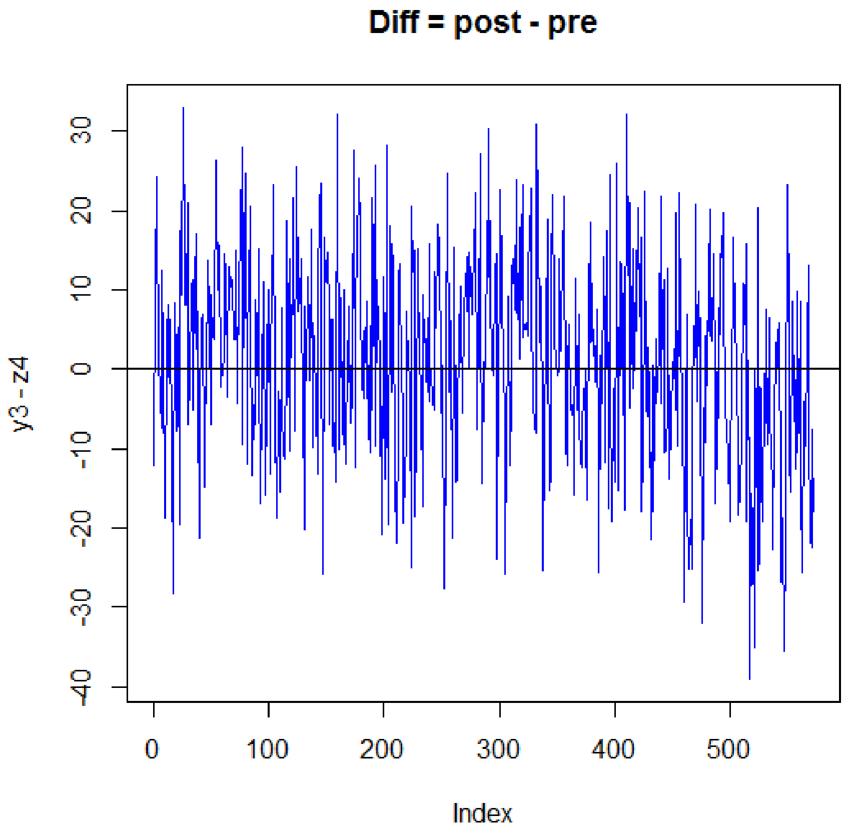
Charissia data

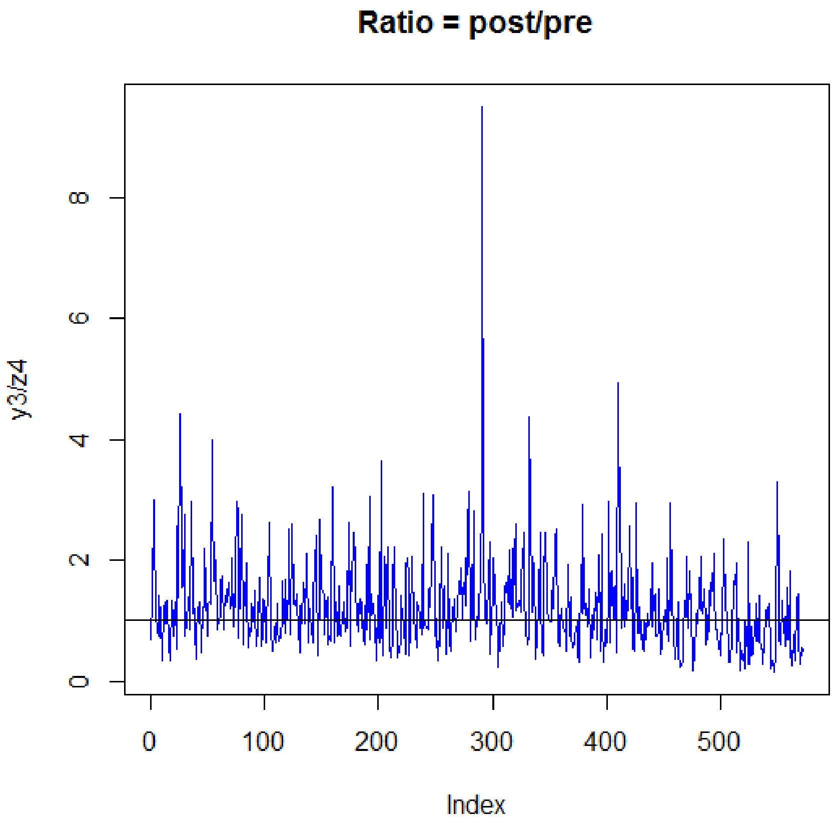
Two time series: pre and post





Compare differences and ratios

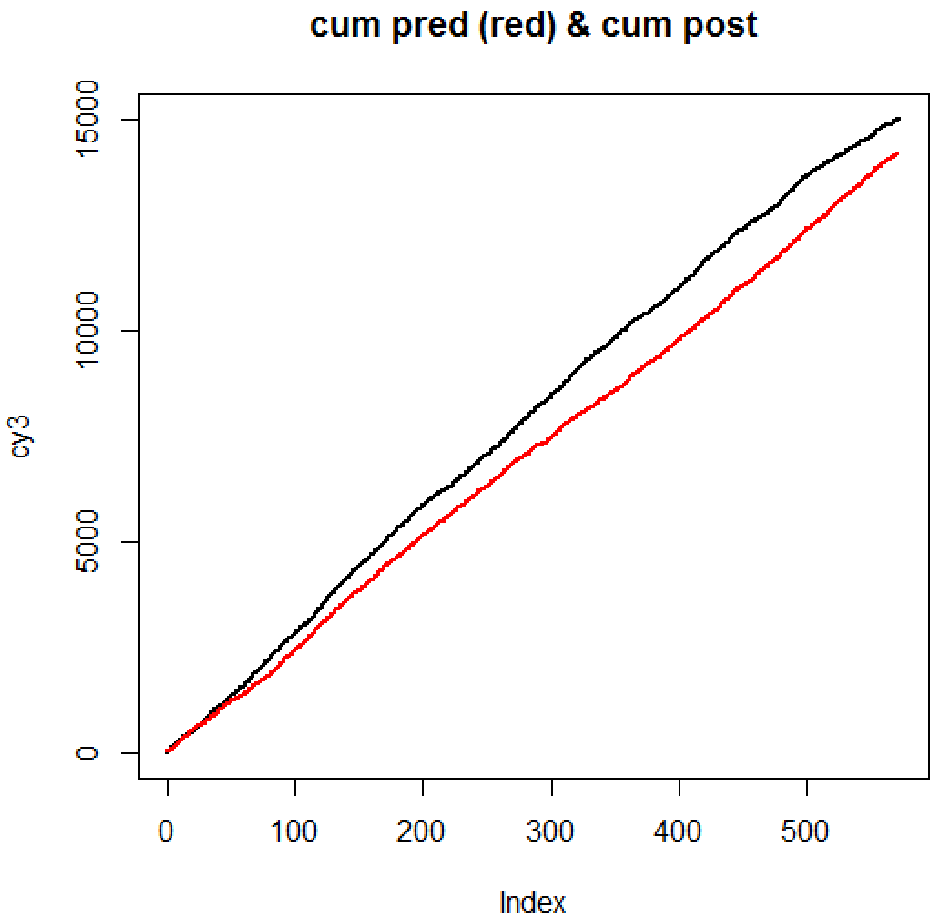




More positive than negative differences in the first graph.

Ratios are much higher above 1.

Compares two cumulative time series.



The post time series has higher values in general.

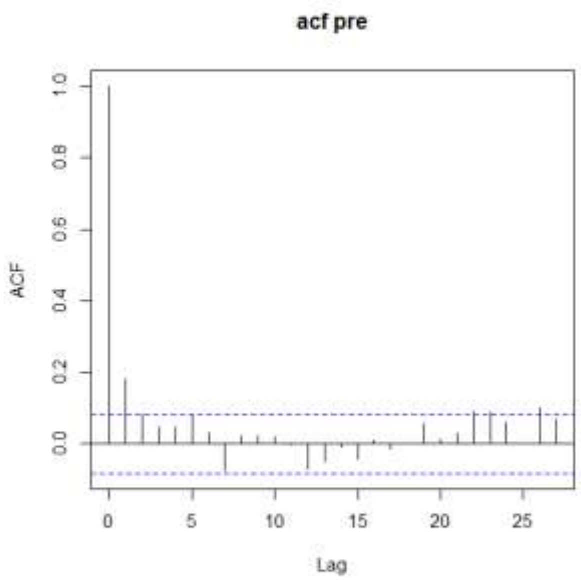
Compare models

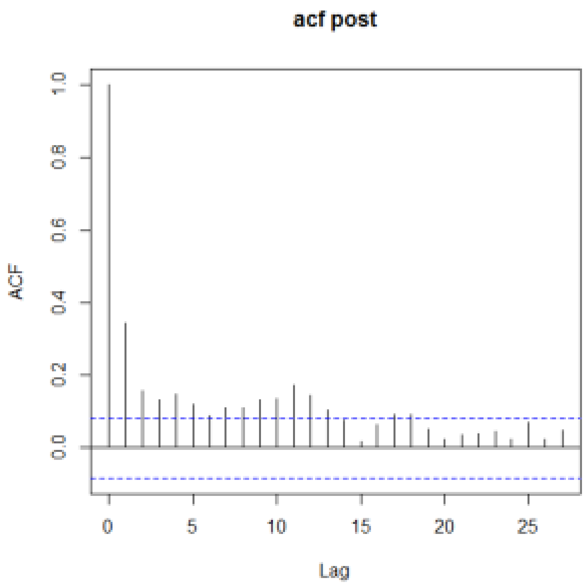
Time series models

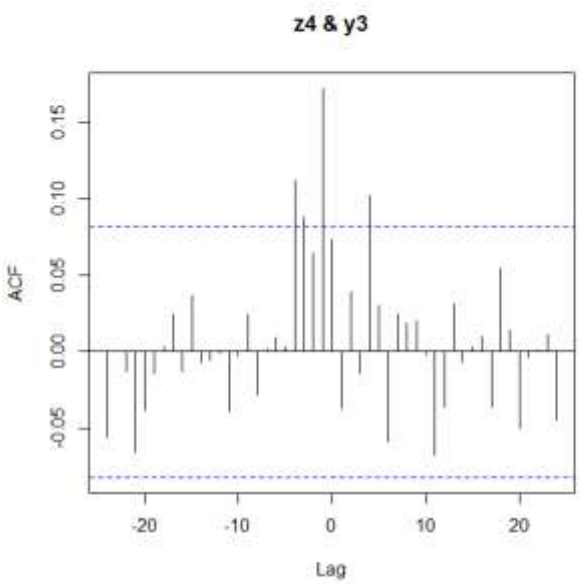
Fix *ARMA* or *regime switching* or *other nonlinear* models and construct an F test to test the hypothesis of a common set of parameters for *a common data set but not here*.

These do not compare mean and variance but just the autocorrelation structure, acf.

The noises are smoothed out.







The three plots are acfs and *cross correlation*. These may not be the comparison Charissia wants!

Running ARMA models

```
arima(x = z4, order = c(1, 0, 1))  #pre

Coefficients:
         ar1      ma1  intercept
      0.5545  -0.3909    24.8376
s.e.  0.2109   0.2356     0.5058

sigma^2 estimated as 78.2:  log likelihood = -2051.22,  aic = 4110.43

arima(x = y3, order = c(1, 0, 1))  #post

Coefficients:
         ar1        ma1         intercept
         0.5976  -0.2983    26.1752
s.e.   0.1648   0.2059     0.6783

sigma^2 estimated as 86.87:  log likelihood = -2088.53,  aic = 4185.07
```

The parameter estimates are similar but there is no test of significance of the difference on *two different data*!!

Compare forecast

Granger test is usually used to see if the values of one series at time *t* can predict the values of the other series at time *t+1,* that is, to test if one time series is useful in forecasting another.

The R code is grangertest() in the lmtest library.

```
> grangertest(z4,y3[1:570],1)  #make two data the same length

Granger causality test

Model 1: y3[1:570] ~ Lags(y3[1:570], 1:1) + Lags(z4, 1:1)
Model 2: y3[1:570] ~ Lags(y3[1:570], 1:1)
  Res.Df Df      F    Pr(>F)
1    566
2    567 -1 14.377 0.0001657 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Significant! But again, it may not be suitable as the time (hence **lag)** pair may differ. We have removed repeats and NA!!!

Compare process in data mining

Distance function is suggested to compare two series in signal processing.

Distance functions:

L-p distance function $D(\mathbf{x},\mathbf{y}) = (\sum|x_i - y_i|^p)^{1/p}$

L-2 distance function $D(\mathbf{x},\mathbf{y}) = (\sum(x_i - y_i)^2)^{1/2}$ (Most popular and efficient)

Why prefer L2 distance?

- L2 distance is preserved under “orthonormal transforms”
  For L-p norm, only p=2 satisfy this property.
- Optimal distance measure for estimation if signals are corrupted by Gaussian, additive noise
- Widely used
- Euclidean distance

Other distance measures

Manhattan distance: $\Sigma_i \, |x_i - y_i \,|$

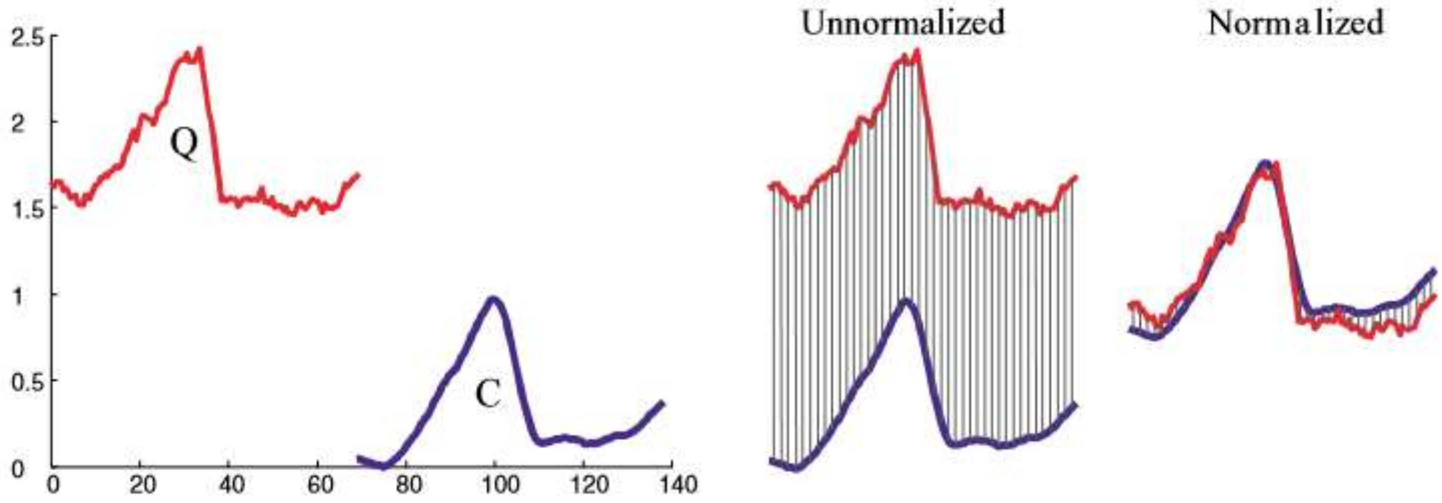
Infinite norm distance: $\max_i |x_i - y_i \,|$

Charissia data

```
> sqrt(sum((z4-y3)^2))
[1] 311.456
```

Again there is no test of significance. It is useful if we have several time series to compare but it depends on the pairing of data.

What constitutes differences?



Difference in time in the first graph?

Align the time series with the same starting point.

Difference in levels in the second graph?

Normalized or standardized the time series.

There are not the issues as the levels of pre and post are similar. The difference in levels is what we want to study.

Elastic dissimilarity measure

What if the time series differ in length? With different sizes M and N!

Dynamic time warping (DTW) $D\left(x_i, y_j\right) = \ \delta\left(x_i, y_j\right) + \min\left\{D_{i,j-1}, D_{i-1,j}, D_{i-1,j-1}\right\}$

where $\delta\left(x_i, y_j\right)$ can be $\left(\sum (x_i - y_i)^2 \right)^{1/2}$.

The unit window can change to any ω in general,

$$j = \max\{1, i' - \omega\}, \cdots, \min\{N, i' + \omega\},$$

where $i' = \left[\frac{iN}{M}\right]$ and $[\cdot]$ is to round to the nearest integer.

If $\omega = 0$ and $M = N$, $D(x, y)$ corresponds to the Euclidean distance. The window ω allows reduction in computational cost.

Minimum jump costs (MJC)

$$D(x, y) = \sum_i c_{\min}^{(i)}$$

where $c_{\min}^{(i)} = \min\left\{c_{t_x}^{t_y}, c_{t_x}^{t_y+1}, c_{t_x}^{t_y+2}, \ldots\right\}$ and $c_{t_x}^{t_y+\Delta} = (\emptyset\Delta)^2 + \delta\left(x_{t_x}, y_{t_{y+\Delta}}\right)$

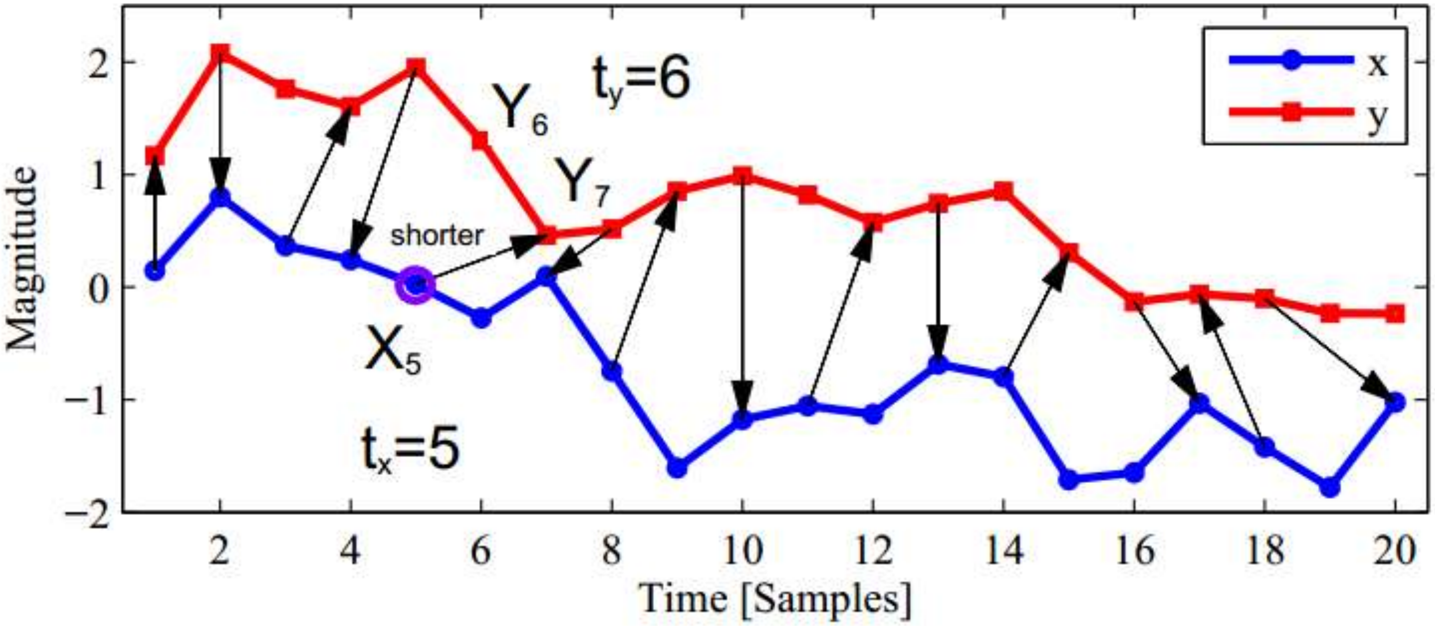


Fig. 1. Example of the recursive jumps performed between time series **x** and **y**. The algorithm starts with time series **x** at $t_x = 1$ (x_1) and ends when $t_x = M$ or $t_y = N$ (x_{20} in the example).

Have jump from y_5 to x_4 (hence $t_x = 5$ and $t_y = 6$) and jump back to y again at minimal cost. Evaluate all possible $t_y + \Delta$, for $\Delta = 0,1,...,14$ (time 6 to 20) and the best jump is y_7.

There are many different distance measures but a paper said the most efficient one is L2, DTW and MJC. However to calculate such distance measures, there is no package or packages are not working.