TUTORIAL EXERCISE PACKAGE - 2013

MATH1015 - BIOSTATISTICS

Semester 1	**Tutorial Week 5**	2013

We encourage you to attempt these problems before your tutorial class.

This week we are dealing with a very important discrete distribution, called The Binomial Distribution. This arises when we have n independent Bernoulli trials each with only two possible outcomes, say "Success" and "Failure". It is assumed that the probability of the "success" is a constant p or $P(S) = p$ and $P(F) = 1 - p$.

If $X \sim \mathrm{B}(n, p)$, then $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}, r = 0, \ldots, n.$

Some binomial probabilities are tabulated in Table 1. Bring a copy of the binomial tables.

Tutorial discussion: Q2, Q3, Q4, Q8 and Q9 marked with *

1. Does X have a binomial distribution in each case below assuming the births are independent with no complications/multiple births and $P(Boy) = P(Girl) = 0.5$? Explain.

(a) Observe the next 15 children born at a local hospital; X is the number of girls among them.

(b) A couple decides to continue to have children until their first girl is born; X is the total number of children in the family.

(c) Each child born to a particular set of parents has probability 0.25 of having blood type O. These parents have 6 children; X is the number of children with blood type O.

2. * If you identify any binomial distribution in Q1, then specify its distribution in the form of $X \sim \mathrm{B}(n, p)$, where n and p are parameters. Write down $P(X = 2)$ and $P(X \leq 2)$ in each binomial situation. Evaluate each probability using the above formula and your calculator.

3. * Suppose that X is binomial $\mathrm{B}(10, 0.4)$. Use binomial tables to find $P(X \leq 4), P(X = 4)$ and $P(X > 3)$.

4. * From previous experiments it is found that 40% of the mice used in an experiment become very aggressive within one minute of being administered an experimental drug. Using the binomial tables, find the probability that, of 10 mice which receive the drug, the number becoming aggressive within one minute is

(a) exactly 4 (b) at most 4 (c) at least 4.

5. Of a large number of mass-produced articles, it is known that one tenth are defective. Writing X for the number of defective items in a random sample of 8 of these articles, explain why X has a binomial distribution.

6. (**Multiple choice**) Writing the distribution of X in Q5 as $X \sim \mathrm{B}(n, p)$, the values of n and p are:

(a) 10 and 8 (b) 0.1 and 8 (c) 8 and 0.1 (d) 8 and 1 (e) ∞ and 5.

7. (**Multiple choice**) Using binomial tables for the binomial random variable X in Q5, $P(X \leq 2)$ is:

(a) 0.9950 (b) 0.8131 (c) 0.7969 (d) 0.9619 (e) -0.9619

8. * Find the mean, variance and standard deviation of X given in Q5.

R questions

9. *Use R to answer Q2 and Q4.
Hint: use dbinom and pbinom in R (see P9 of Week 5 notes for details).

10. Conduct a simulation experience to generate 100 random variables X which follow a binomial distribution with $n = 5$ and $p = 0.6$. The command in R is

```
set.seed(1235)
x=rbinom(100,5,0.6)
x
```

(a) Obtain the table of observed frequencies for `x` using `table(x)` and plot the distribution using `hist(x, breaks=c(-1,0,1,2,3,4,5))`.

(b) Find the mean $\bar{x}$ and variance s^2 of `x` using `mean(x)` and `var(x)` respectively. Are they close to $E(X) = np$ and $Var(X) = np(1 - p)$ respectively? Note that the two values are closer if we generate more `x`, e.g. 200 instead of 100.

(c) Obtain a probability distribution of X using

```
r=c(0,1,2,3,4,5)
pr=dbinom(r,5,0.6)
rbind(r,pr)
```

Check the mean using `sum(r*pr)`. Does it equal exactly to $E(X) = np$?

1. Each child born to a particular set of parents has probability 0.3 of having blood type O. These parents have 6 children and let X be the number of children with blood type O. Using the binomial tables, find $P(X \leq 2)$.

2. Let $X \sim \mathrm{B}(5, 0.3)$. Complete the following table:

i	0	1	2	3	4	5
p_i	*	*	*	*	*	*

Find E(X) and Var(X). Verify that E(X) = np and Var(X) = np(1-p).

3. A production line is working satisfactorily if at most 10% of items are faulty. To monitor quality control, a random sample of 10 items is inspected once a week. The production manager orders the line to be shut down for repairs if 3 or more of the items in the sample are faulty.

(a) Use tables to find the probability that the line is shut down in a particular week during which exactly 10% of the items produced are faulty. (You may assume independence.)

(b) Repeat (a) when the true percentage of faulty items is

(*i*) 20%, (*ii*) 30%, (*iii*) 40%, (*iv*) 50%, (*v*) 60%.

4. If X is binomial $B(5, 0.2)$, use the binomial tables to complete this table, writing $p_i = P(X = i)$. Find E(X) using this table.

i	0	1	2	3	4	5	Total
p_i							1.0000