Semester 1	**Tutorial Week 6**	2013

You need copies of **binomial** and **normal** tables to do this tutorial. These tables are available on the course web.

If $X \sim B(n,p)$ then $E(X) = np$ and $Var(X) = np(1-p)$ (A)

If $X \sim B(n,p)$ and $Y \sim B(n, 1-p)$ then $P(X = k) = P(Y = n-k)$ (B)

If $X \sim N(\mu, \sigma^2)$ then $Z = \dfrac{X-\mu}{\sigma}$ is standard normal or $Z \sim N(0,1)$ (C)

Tutorial discussion: Q4, Q7, Q8 and Q9 marked with *

1. Let $X \sim B(10, 0.3)$ be a binomial random variable. Find $E(X)$, $Var(X)$ and $SD(X)$. **Hint: use (A).**

2. Suppose that $X \sim B(15, 0.6)$. Find $P(X = 13)$ and $P(X = 5)$. Without using the calculator, deduce the values of $P(Y = 2)$ and $P(Y = 10)$ when $Y \sim B(15, 0.4)$. **Hint: use (B).**

3. (**Multiple choice**) Let $X \sim N(2, 3^2)$. This tells us that the distribution of X is:

(a) Binomial with mean 2 and variance 9
(b) Binomial with $n = 2$ and $p = 9$
(c) Normal with mean 2 and variance 3
(d) Normal with mean 2 and variance 9
(e) Normal with mean 0 and variance 9

4. * Let $Z \sim N(0,1)$ be a standard normal distributed r.v. Find

(a) $P(Z < 1.2)$
(b) $P(Z > -1.2)$
(c) $P(Z < -1.2)$
(d) $P(-1.2 < Z < 1.2)$
(e) $P(-0.81 < Z < 2.32)$
(f) a such that $P(Z < a) = 0.67$
(g) b such that $P(-b < Z < b) = 0.8$

5. Write down the standardised version, Z of the random variable X in Q3. **Hint: use (C).**

6. Let X be the random variable in Q3. Complete the following:
$P(X > 8) = P(Z >$ __________ $) =$ __________ .

7. * If $X \sim N(10, 16)$, find

(a) $P(X > 12)$
(b) k such that $P(X \leq k) = 0.90$

8. * The number of hours per week spent watching TV by a young child is approximately normal with mean, $\mu = 15$ hours and standard deviation, $\sigma = 2$ hours. Find the probability that a child (chosen at random) watch TV in a week (i) between 15 and 19 hours (ii) at least 16 hours (iii)at most 14 hours. Compare your answers with table 3 of the book.

Use R to answer Q7, Q8 and Q9. See instructions below.

9. * The `hospital` data of discharged patients contains the following columns:

```
Column  Label
1       ID no.
2       Duration of hospital stay
3       Age
4       Sex  1=male  2=female
5       First temperature following admission
6       First WBC(x1000) following admission
7       Received antibiotic 1=yes  2=no
8       Received bacterial culture 1=yes  2=no
9       Service 1=med 2=surg.
```

Read the data using

```
dat=read.table(file=url("http://www.maths.usyd.edu.au/math1015/r/hospital.txt"),skip=1)
```

Set `stay=dat[,2]` and `age=dat[,3]` to be vectors of duration of hospital `stay` and `age` respectively.

(a) Plot histograms of `stay` and `age` using 10 breaks. Explain why the distribution of `age` can be better approximated by a normal distribution. **Hint:** use `hist(stay,breaks=10)` for `stay`.

(b) Find the sample mean and sample s.d. of `age`.

(c) Assuming that the distribution of `age` is normal and using results of (b) to estimate the mean and s.d., find the probability that a randomly chosen discharged patient is older than 60 years of age.

For binomial calculations: Use `dbinom(x,n,p)` and `pbinom(x,n,p)` to find respectively, the individual probability $\mathrm{P}(X = \mathtt{x})$ and cumulative probability $\mathrm{P}(X \leq \mathtt{x})$ if $X \sim \mathrm{B}(\mathtt{n},\mathtt{p})$.

For normal calculations: Use `pnorm(x,m,s)` and `qnorm(p,m,s)` to find respectively, the cumulative probability $\mathrm{P}(X < \mathtt{x})$ and quantile a such that $\mathrm{P}(X < a) = \mathtt{p}$ if $X \sim \mathrm{N}(\mathtt{m}, \mathtt{s}^2)$.

1. Let $Z \sim \mathcal{N}(0, 1)$.

(a) Find

(*i*) $P(-1.72 < Z < 0.52)$ (*ii*) $P(|Z| > 1.96)$.

(b) Find the values of a, b, and c such that

(*i*) $P(Z \leq a) = 0.7291$ (*ii*) $P(Z > b) = 0.10$ (*iii*) $P(|Z| < c) = 0.90$.

2. Suppose that X, the breaking strength of rope in pounds , has distribution $\mathcal{N}(100, 16)$. Each 30 metre coil of rope brings a profit of \$25 provided $X > 95$. If $X \leq 95$ then the rope is used for a different purpose and a profit of only \$10 per coil is made. Find the average profit per coil.

3. Glaucoma is a disease of the eye that is manifested by high intraocular pressure. The distribution of intraocular pressure in unaffected adults is approximately normal with mean 16 mm Hg and standard deviation 4 mm Hg.

(a) If the normal range for intraocular pressure (in mm Hg) is considered to be 12 – 20, what percentage of unaffected adults would fall within this range?

(b) An adult is considered to have *abnormally high* intraocular pressure if the pressure reading is in the top percentile for unaffected adults. Use tables to state pressures considered to be abnormally high.

4. Check the answers in Q1 to Q3 above using R.