Semester 1 **Tutorial Week 7** 2013

If $X_1, X_2, \cdots, X_n$ is a random sample from $N(\mu, \sigma^2)$ then $\bar{X}$ is $N(\mu, \frac{\sigma^2}{n})$, where $\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$ is the sample mean.

CLT: Let $X_1, X_2, \cdots, X_n$ be a random sample from a population with mean μ and variance σ^2. Then for large n, the sample mean $\bar{X}$ is approximately $N(\mu, \frac{\sigma^2}{n})$.

Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal distribution. The $(1-\alpha)\%$ CI for the population mean μ is $\left(\bar{X} - t_{n-1,\alpha/2}\, \frac{S}{\sqrt{n}},\ \bar{X} + t_{n-1,\alpha/2}\, \frac{S}{\sqrt{n}}\right)$.

Let $X_1, X_2, \cdots, X_n$ be a large sample from a population of binary variables. The $(1-\alpha)\%$ CI for the population proportion p is $\left(\hat{p} - Z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + Z_{1-\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$ where $\hat{p} = X/n$ is the sample proportion.

Tutorial discussion: Q5, Q9, Q10 and Q11 marked with *

1. Multiple choice Suppose that $X_i \sim B(50, 0.02)$. The distribution of sample mean $\bar{X}$ based on a random sample of size $n = 100$ is approximately:

(a) $N(50, 0.02)$ (b) $N(50, 1)$ (c) $N(1, 0.98)$ (d) $N(1, 0.0098)$ (e) $N(0.01, 0.098)$

2. Multiple choice Suppose that $X_1, X_2, \cdots, X_{16}$ is a random sample of size 16 (< 30) from the distribution $N(100, 25)$. The distribution of $\bar{X}$ (the sample mean) is:

(a) $\mathcal{N}(100, 25)$ (b) $\mathcal{N}(100, \frac{5}{4})$ (c) $\mathcal{N}(0, 25)$ (d) $\mathcal{N}(100, \frac{25}{16})$ (e) $\mathcal{N}(0, 1)$

3. Multiple choice Professor Bigbeg wants to calculate, $p = P(\bar{X} \le 102)$ in Q2. He told his students to standardise $\bar{X}$ and write, $p = P(Z \le z)$ (Z is $\mathcal{N}(0, 1)$). The value of z is:

(a) 102 (b) 2 (c) 32/25 (d) 0 (e) 8/5

4. Find $P(\bar{X} \le 102)$ in Q3 using the standard normal table.

5. *The number of calories in a salad on the lunch menu is approximately normally distributed with mean of 200 and standard deviation of 5. Find the probability that a randomly chosen plate will contain more than 208 calories. Ryan orders 16 dishes for a meeting. Find the probability that the average number of calories in this sample will exceed 208. Interpret this last probability.

6. Let $X_1, X_2, \cdots, X_n$ be a random sample from $N(\mu, \sigma^2)$. State the distribution of:

(a) $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ (σ is known).

(b) $t = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$ (σ is unknown and S is an estimator of σ).

7. A data set of 27 values gave the sample variance of $s^2 = 0.00639$. What is the standard error (se) of the sample mean for this sample?

8. Find k such that $P(-k < t_{15} < k) = 0.90$.

9. *A sample of 16 observations from a normal population gave $\bar{x} = 194.8$ and $s = 13.14$.
(a) Find a 90% CI for the mean, μ. Interpret this CI.

(b) Find a 95% CI for μ.

Use R to answer Q10 and Q11.

10. *Use R to answer Q5 and Q8.

Hint for normal and t distributions:

If $X \sim N(m, s^2)$, use `pnorm(k,m,s)` to find the cumulative probability $P(X < k)$.
If $X \sim N(m, s^2)$, use `qnorm(p,m,s)` to find the quantile k such that $P(X < k) = p$.
If $T \sim t_{n-1}$, use `pt(k,n-1)` to find the cumulative probability $P(T < k)$.
If $T \sim t_{n-1}$, use `qt(p,n-1)` to find the quantile k such that $P(T < k) = p$.

11. *Read the germination data from the file `germination.txt` in R using

```
x = scan(file=url("http://www.maths.usyd.edu.au/math1015/r/germination.txt"))
```

Find the mean and the sd of this data. Find a 95% CI for μ using R.

Hint for CI: Read the data using the buil-in function `t.test(x)` and ignore the part of the output which doesn't involve the CI. Output will contain a 95% CI for μ. (These steps will be explained later in week8).

Semester 1	Problem Sheet Week 7	2013

1. The mean and standard deviation of all contributions to a major charity in a year are $\mu = \$69$ and $\sigma = \$48$. If a random sample of 100 contributions is selected, state the mean and variance of the distribution of the sample mean, $\bar{X}$. Compute the probability that the sample mean will

(a) exceed \$80 (b) lie within \$10 from μ (ie.$P(59 \le \bar{X} \le 79)$.

2. A new type of electronics flash for cameras will last an average of 5000 hours with a standard deviation of 500 hours. A company quality control engineer intends to select a random sample of 100 of these flashes and use them until they fail. What is the probability that the mean life time of 100 flashes will be
(a) less than 4928 hours

(b) between 4950 and 5050 hours (ie. within 50 hours of μ, the population mean)

3. It is known that the distribution of the weights (in kg) of male students follows $\mathcal{N}(70, 10^2)$. What is the probability that the average weight of 16 randomly chosen male students will exceed 75kg?

4. Book P.109, Q2

5. Book P.110, Q8.