Semester 1 | Solution to Tutorial 3 | 2013

1. Ans: D

$$\begin{aligned}\bar{x} &= \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{427}{8} = 53.375\\ s^2 &= \frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right] = \frac{1}{8-1}\left[22805 - \frac{427^2}{8}\right] = 1.982143\end{aligned}$$

2. Ans: C

x_i	1	6	8	3	5	Total
f_i	3	1	7	4	2	17
$f_i\,x_i$	3	6	56	12	10	87
$f_i\,x_i^2$	3	36	448	36	50	573

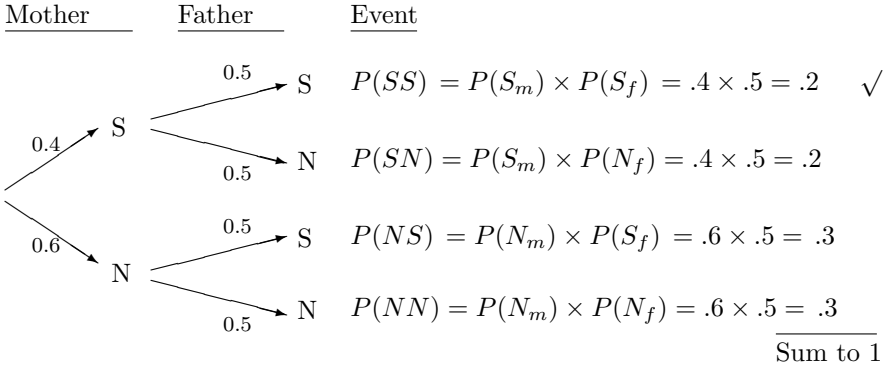
$$\bar{x} = \frac{1}{17}(1\times 3 + 6\times 1 + 8\times 7 + 3\times 4 + 5\times 2) = 5.117647$$

3.
$$s^2 = \frac{1}{n-1}\left[\sum_{i=1}^{5} f_i\,x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{5} f_i\,x_i\right)^2\right] = \frac{1}{17-1}\left[573 - \frac{87^2}{17}\right] = 7.985294$$

4. Ans: e. Since the event is $A = \{(1,1),(1,2),(2,1)\}$. Hence the probability, $P(A) = \dfrac{3}{36} = \dfrac{1}{12}$.

5. (a) $P(CAB) = \frac{1}{3}\times\frac{1}{2} = \frac{1}{6}$; (b) $P(B..) = \frac{1}{3}$; (c) $P(..C) = \frac{1}{3}$; (d) $P(..A) = \frac{1}{3}$ and hence $P(..A^C) = \frac{2}{3}$

6. The probability tree is



The probability that both mother and father are smokers is $0.4 \times 0.5 = 0.2$

7. (a) $B = \{(1,2),(2,1),(1,5),(5,1),(2,4),(4,2),(3,3),(3,6),(6,3),(4,5),(5,4),(6,6)\}$.
   (b) Given B there are only two possibilities of observing the elements of A. Thus $P(A|B) = \frac{2}{12} = \frac{1}{6}$.
   (c) $A \cap B = \{(1,2),(2,1)\}$ and $P(A\cap B) = \frac{2}{36} = \frac{1}{18}$ and $P(B) = \frac{12}{36} = \frac{1}{3}$. These give $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1/18}{1/3} = \frac{1}{6}$.

8. R exercise

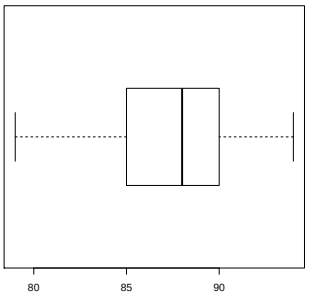
```
> x=scan(file=url("http://www.maths.usyd.edu.au/math1015/r/germination.txt"))
> fivenum(x)  #(ii)
[1] 79 85 88 90 94
> boxplot(x) #(iii)
> stem(x)  #(iv)

  The decimal point is at the |

  78 | 0
  80 | 00
  82 | 00
  84 | 000000000000
  86 | 00000000000000
  88 | 0000000000000000000
  90 | 00000000000
  92 | 00000
  94 | 0

> mean(x)/100  #(v)
[1] 0.8758462
> length(x[x==90])/length(x)  #(vi)
[1] 0.0923077
> length(x[x<=79])/length(x)  #(vii)
[1] 0.01538462
```

(iii)



(iv) Quite consistent. Both gives distributions skewed to the left.

9. `> dat=read.table(file=url("http://www.maths.usyd.edu.au/math1015/r/hospital.txt"),skip=1)`

```
> dat
   V1 V2 V3 V4   V5 V6 V7 V8 V9
1   1  5 30  2 99.0  8  2  2  1
2   2 10 73  2 98.0  5  2  1  1
3   3  6 40  2 99.0 12  2  2  2
4   4 11 47  2 98.2  4  2  2  2
5   5  5 25  2 98.5 11  2  2  2
6   6 14 82  1 96.8  6  1  2  2
7   7 30 60  1 99.5  8  1  1  1
8   8 11 56  2 98.6  7  2  2  1
9   9 17 43  2 98.0  7  2  2  1
10 10  3 50  1 98.0 12  2  1  2
```

```
11 11  9 59  2 97.6  7  2  1  1
12 12  3  4  1 97.8  3  2  2  2
13 13  8 22  2 99.5 11  1  2  2
14 14  8 33  2 98.4 14  1  1  2
15 15  5 20  2 98.4 11  2  1  2
16 16  5 32  1 99.0  9  2  2  2
17 17  7 36  1 99.2  6  1  2  2
18 18  4 69  1 98.0  6  2  2  2
19 19  3 47  1 97.0  5  1  2  1
20 20  7 22  1 98.2  6  2  2  2
21 21  9 11  1 98.2 10  2  2  2
22 22 11 19  1 98.6 14  1  2  2
23 23 11 67  2 97.6  4  2  2  1
24 24  9 43  2 98.6  5  2  2  2
25 25  4 41  2 98.0  5  2  2  1
> stay=dat[,2]
> stay
 [1]  5 10  6 11  5 14 30 11 17  3  9  3  8  8  5  5  7  4  3  7  9 11 11  9  4
> sex=dat[,4]
> stay.f=stay[sex==2]
> stay.f
 [1]  5 10  6 11  5 11 17  9  8  8  5 11  9  4
> mean(stay.f)   #(a)
[1] 8.5
> length(stay.f)/length(stay)  #(b)
[1] 0.56
> length(stay.f[stay.f>=8])/length(stay.f)  #(c)
[1] 0.6428571
```

Problem set - Week3

1. Ans: A

x_i	2	4	7	10	12	Total
f_i	1	3	6	8	11	29
$f_i\,x_i$	2	12	42	80	132	268

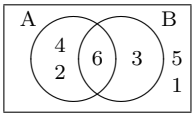
$$\bar{x} = \frac{1}{29}(2\times 1 + 4\times 3 + 7\times 6 + 10\times 8 + 12\times 11) = 9.241379$$

2. The event is $\{1, 2, 3\}$.
   The probability of an even no. is $\frac{2}{9}$.
   The probability of an odd no. is $\frac{1}{9}$.

$$P(\text{the no.} < 4) = P(1) + P(2) + P(3) = \frac{1}{9}\times 2 + \frac{2}{9} = \frac{4}{9}$$

3. We have

$$\begin{aligned} A &= \{2,4,6\},\ B = \{3,6\},\ S = \{1,2,3,4,5,6\}, \\ A\cap B &= \{6\},\ A\cup B = \{2,3,4,6\}, \end{aligned}$$



$$\begin{aligned} P(A \cap B) &= P(6) = \frac{2}{9} \\ P(A \cup B) &= P(2,3,4,6) = \frac{1+3\times 2}{9} = \frac{7}{9} \\ &= P(A) + P(B) - P(A \cap B) = \frac{6}{9} + \frac{3}{9} - \frac{2}{9} = \frac{7}{9} \end{aligned}$$

A and B are not m.e. since $A \cap B$ is not empty.

4. A die has 2 faces W, 1 face R and 3 faces G and is thrown 3 times.
   (a) $P(\text{white}) = \frac{2}{6} = \frac{1}{3}$
   (b) $P(\text{same colour}) = P(WWW) + P(RRR) + P(GGG) = \left(\frac{1}{3}\right)^3 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{2}\right)^3 = 0.167$