Semester 1	Solution to Tutorial 4	2013

1. D

   Solution: Denote the events $A_i = \{$the man hits the target on the ith attempt$\}$, $i = 1, 2, 3, 4$.

   The target will be hit when the man hits it once or twice or thrice or all four times in the four shots that he takes. That is, he is not missing at all four attempts.
   P(missing all four attempts)= $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{4}\right)\left(1-\frac{1}{4}\right) = (\frac{3}{4})^4 = \frac{81}{256}$.

   Therefore, the required probability is $1 - \frac{81}{256} = \frac{175}{256}$.

2. **Answer:** e

   Solution: Let p be the probability of observing an odd number. Then the probability of observing an even number is $2p$. Since the probabilities of observing 1, 2, ..., 6 must add up to 1, we have:
   $p + 2p + p + 2p + p + 2p = 9p = 1 \implies p = \frac{1}{9}$

   If we roll the die two times, then the event $\{\text{Sum} = 5\} = \{(1, 4), (4, 1), (2, 3), (3, 2)\}$.
   The probability of each of these outcomes is $\frac{1}{9}\left(\frac{2}{9}\right) = \frac{2}{81}$. Therefore,
   P(Sum =5) $= 4\left(\frac{2}{81}\right) = \frac{8}{81}$.

3. * **Tutors, please note that the students have not seen the notation** $\binom{5}{2} = 5_{C_2}$ **for combinations. Therefore, in Q3, Q4 and Q5 exalain all possibilities.**

   (a) $P(\text{same exit}) = 5\left(\frac{1}{5}\right)^4 = 0.008$.

   (b) There are 10 ways of selecting two doors from 5 (doors 12, 13, 14, 15, 23, 24, 25, 34, 35, 45) and each way has 2 choices of assigning one door to A,B,C and the other door to D. Therefore,
   $P(\text{A,B,C the same \& D different}) = 2 \times 10 \times \left(\frac{1}{5}\right)^4 = 0.032$.

   (An alternative, simple argument is: Three of them come out at the same exit with probability $(1/5)^3$ and there are 4 such choices (one door for D). Hence the required probability is 4/125.

4. $P(\text{all tall}) = \left(\frac{3}{4}\right)^5 = 0.2373047$.

5. *

   (a) The number of tall offsprings in a random sample of 5 can be 0,1,2,3,4,5. Hence X is discrete valued.

   (b) When there are all short or tall offsprings, that is $X = 0$ and $X = 5$ respectively, the probabilities are

   $$P(X=0) = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}; \quad P(X=5) = \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$$

   When $X = 1$, the combinations are

   TSSSS, STSSS, SSTSS, SSSTS, SSSST

So there are 5 combinations totally and each combination occurs at the same probability. Similarly, when $X = 4$, the number of combinations are still 5 because we can simply swap S and T. Hence the probabilities are

$P(X = 1) = 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) = \frac{15}{1024}$; $P(X = 4) = 5\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^4 = \frac{405}{1024}$

When $X = 2$, the combinations are

TTSSS, TSTSS, TSSTS, TSSST, STTSSS, STSTS, STSST, SSTTS, SSTST, SSSTT

So there are 10 combinations totally and each combination occurs at the same probability. Similarly, when $X = 3$, the number of combinations are still 10 because we can simply swap S and T. Hence the probabilities are

$P(X = 2) = 10\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = \frac{90}{1024}$; $P(X = 3) = 10\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3 = \frac{270}{1024}$

Hence

x	0	1	2	3	4	5
$P(X = x)$	$\frac{1}{1024}$	$\frac{15}{1024}$	$\frac{90}{1024}$	$\frac{270}{1024}$	$\frac{405}{1024}$	$\frac{243}{1024}$

6. * Since $\sum_{i=0}^{5} p_i = 1$, the missing entry is $1 - 0.17 - 0.36 - 0.31 - 0.03 = 0.13$.

   (a) $\mathrm{E}(X) = 0 \times 0.17 + 1 \times 0.36 + 2 \times 0.31 + 3 \times 0.13 + 4 \times 0.03 = 1.49$.

   (b) $\mathrm{E}(X^2) = 0^2 \times 0.17 + 1^2 \times 0.36 + 2^2 \times 0.31 + 3^2 \times 0.13 + 4^2 \times 0.03 = 3.25$.

   (c) $\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = 3.25 - 1.49^2 = 1.03$.

7. $\mathrm{E}(X) = 0 \times 0.05 + 1 \times 0.10 + 2 \times 0.30 + 3 \times 0.40 + 4 \times 0.15 = 2.50$. This means that on average there are 2.5 colonies per dish.

8. `R`:

   (a) Read the data using

```
x = read.table(file=url("http://www.maths.usyd.edu.au/math1015/r/
IronRetained.txt"), skip=3)
```

   (b) Percentages are increasing for each level in Fe^{2+} and Fe^{3+}. Fe^{3+} percentages are higher than Fe^{2+} concentrations.

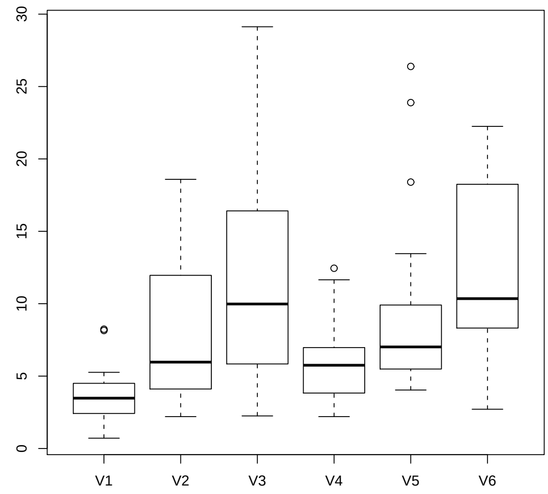
```
mean(x)

       V1          V2          V3          V4          V5          V6
3.698889  8.203889 11.750000  5.936667  9.632222 12.639444

apply(x, 2, median)

    V1     V2     V3     V4     V5     V6
 3.475  5.965  9.980  5.750  7.015 10.350
```

   (c) `boxplot(x)`



Again, we see that the median levels go up with each level. It also seems that variability increases with each level. The 0.3 concentration produces right-skewed distributions. There were three outliers in the case Fe^{3+} with concentration 1.2.

9. Read the data and set column 7 and 8 to be `anti` and `bac` respectively.

```
dat = read.table(file=url("http://www.maths.usyd.edu.au/math1015/r/
                           hospital.txt"),skip=1)
anti=dat[,7]
bac=dat[,8]
```

(a) Count the total number of observations and the number of observations such that `anti=1` and `bac=1`.

```
n=length(anti)
n.both=length(anti[anti==1 & bac==1])
c(n.both,n)
[1]  2 25
p.both=n.both/n
[1] 0.0800
```

Then the probability of receiving both treatments is $P(AB) = \frac{\#(AB)}{n} = \frac{2}{25} = 0.08$. Since $P(AB) \neq 0$, the 2 events A and B are NOT *mutually exclusive*.

(b) Count the number of observations such that `anti=1` and the number of observations such that `bac=1`.

```
n.anti=length(anti[anti==1])
n.bac=length(bac[bac==1])
c(n.anti,n.bac)
[1]  7  6
```

Then the probability of A is $P(A) = \frac{\#(A)}{n} = \frac{7}{25} = 0.28$
and the probability of B is $P(B) = \frac{\#(B)}{n} = \frac{6}{25} = 0.24$. Since

$$P(A)P(B) = \frac{7}{25} \times \frac{6}{25} = 0.0672 \neq 0.08 = P(AB),$$

events A and B are not *independent*.

```
p.anti=n.anti/n
p.bac=n.bac/n
c(p.anti,p.bac,p.anti*p.bac)
[1] 0.2800 0.2400 0.0672
```

Problem set - Week 4

1. There are 36 possible outcomes as follows:

(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

These are 36 outcomes are equally likely and therefore,

(a) $\frac{6}{36} = \frac{1}{6}$;

(b) $\frac{10}{36} = \frac{5}{18}$;

(c) $\frac{18}{36} = \frac{1}{2}$.

2.

$$\begin{aligned} E(X) &= 1 \times .35 + 2 \times .30 + 3 \times .25 + 4 \times .10 = 2.1 \\ E(\frac{1}{X}) &= 1 \times .35 + \frac{1}{2} \times .30 + \frac{1}{3} \times .25 + \frac{1}{4} \times .10 = 0.6083 \\ E(X^2) &= 1^2 \times .35 + (2)^2 \times .30 + (3)^2 \times .25 + (4)^2 \times .10 = 5.4 \\ Var(X) &= E(X^2) - [E(X)]^2 \\ &= 5.4 - (2.1)^2 \\ &= 0.99 \end{aligned}$$

3. Expected number of colonies after spraying is

$0 \times 0.21 + 1 \times 0.43 + 2 \times 0.21 + 3 \times 0.12 + 4 \times 0.03 = 1.33$.

Thus the expected number of colonies is reduced by $2.50 - 1.33 = 1.17$