Semester 1	Solution to Tutorial Set 5	2013

1. (a) X is binomial, since there are only two possibilities at each birth (i.e. boy or girl), with a fixed number of independent trials ($n = 15$) and a fixed success probability ($p = 0.5$)in each trial.

   (b) X is *not* binomial, since the number of trials, n is not fixed

   (c) X is binomial, since each has blood type O or not O, with a fixed number of independent trials($n = 6$) and a fixed success probability ($p = 0.25$)in each trial.

2. In (a), $X \sim B(15, 1/2)$ and the distribution is symmetric since $p = \frac{1}{2}$.
   In (c), $X \sim B(6, 0.25)$ and the distribution is not symmetric since $p \neq \frac{1}{2}$.

$$\begin{aligned}
\text{(a) } P(X = 2) &= \binom{15}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{13} = \binom{15}{2}\left(\frac{1}{2}\right)^{15} = 0.0032 \\
P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\
&= \binom{15}{0}\left(\frac{1}{2}\right)^{15} + \binom{15}{1}\left(\frac{1}{2}\right)^{15} + \binom{15}{2}\left(\frac{1}{2}\right)^{15} = 0.0037. \\
\text{(c) } P(X = 2) &= \binom{6}{2}(0.25)^2(0.75)^4 = 0.2966; \\
P(X \leq 2) &= \binom{6}{0}(0.75)^6 + \binom{6}{1}(0.25)(0.75)^5 + \binom{6}{2}(0.25)^2(0.75)^4 = 0.8306.
\end{aligned}$$

3. $n = 10$ and $p = 0.4$.

$$\begin{aligned}
P(X \leq 4) &= 0.6331 \\
P(X = 4) &= 0.6331 - 0.3823 = 0.2508 \\
P(X > 3) &= 1 - P(X \leq 3) = 1 - 0.3823 = 0.6177
\end{aligned}$$

4. Let X be the number of mice becoming aggressive within one minute.

$$X \sim B(10, 0.4) \quad \text{(if each trial is independent)}$$

   Show how this is related to Q3.
   (a) $P(X = 4) = 0.2508$; (b) $P(X \leq 4) = 0.6331$; (c) $P(X \geq 4) = P(X > 3) = 0.6177$

5. In this experiment the number of trials is fixed, each independent trial consists of defective/nondefective (or success/failure) outcomes, and the probability of a defective at each trial is the same. Therefore, the distribution of X is binomial.

6. Ans (c) since $n = 8$ & $p = 0.1$.

7. Ans (d) since $P(X \leq 2) = 0.9619$.

8. From (6), we have $n = 8$ and $p = 0.1$.

$$\begin{aligned} \mathrm{E}(X) &= np = 8 \times 0.1 = 0.8 \\ \mathrm{Var}(X) &= np(1-p) = 8 \times 0.1 \times 0.9 = 0.72 \\ \mathrm{SD}(X) &= \sqrt{0.72} = 0.8485 \end{aligned}$$

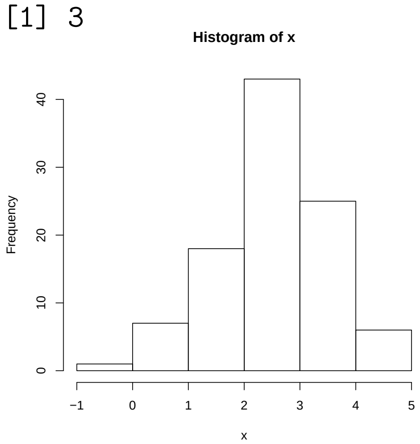
Answers to R problems:

9. R for Q2: (a) `dbinom(2, 15, 0.5);   pbinom(2, 15, 0.5);`

   (c) `dbinom(2, 6, 0.25);   pbinom(2, 6, 0.25);`

   R for Q4: `pbinom(4, 10, 0.4);   dbinom(4, 10, 0.4);  1 - pbinom(3, 10, 0.4)`

10.
```
> set.seed(1235)  #set a seed to generate 100 values
> x=rbinom(100,5,0.6)
> x
  [1] 4 3 4 2 2 3 3 2 3 4 1 3 3 3 2 5 1 3 3 2 4 1 2 4 4 2 4 3 4 3 3 3 2 3 4 3 4
 [38] 4 3 4 3 2 5 3 2 2 4 3 2 4 3 3 4 4 3 3 4 1 3 5 4 3 4 3 1 3 4 2 1 4 2 3 3 4
 [75] 3 2 5 3 1 5 3 3 4 3 3 2 3 2 3 3 3 5 4 3 3 4 2 3 0 3
> table(x)  #(a)
x
 0  1  2  3  4  5
 1  7 18 43 25  6
> hist(x, breaks=c(-1,0,1,2,3,4,5))   #(a)
> mean(x)  #(b)
[1] 3.02
> var(x)   #(b)
[1] 1.050101
> 5*0.6  #true mean
[1] 3
> 5*0.6*(1-0.6)  #true var
[1] 1.2
> r=c(0,1,2,3,4,5)  #(c)
> pr=dbinom(r,5,0.6)  #(c)
> rbind(r,pr)   #(c)
      [,1]   [,2]   [,3]   [,4]   [,5]    [,6]
r  0.00000 1.0000 2.0000 3.0000 4.0000 5.00000
pr 0.01024 0.0768 0.2304 0.3456 0.2592 0.07776
> sum(r*pr)  #(c)
[1] 3
```



Problem Set 5

1. $P(X \leq 2) = 0.7443$.

2.

i	0	1	2	3	4	5	Total
p_i	0.1681	0.3601	0.3087	0.1323	0.0284	0.0024	1.0000

Use `dbinom(i,5,0.3)`, i=0,1,2,3,4,5.

$E(X) = 0 \times 0.1681 + 1 \times 0.3601 + 2 \times 0.3087 + 3 \times 0.1323 + 4 \times 0.0284 + 5 \times 0.0024 = 1.5.$

$$\begin{aligned} Var(X) &= E(X^2) - [E(X)]^2 \\ &= 0^2 \times 0.1681 + 1^2 \times 0.3601 + 2^2 \times 0.3087 + 3^2 \times 0.1323 + 4^2 \times 0.0284 + 5^2 \times 0.0024 - 1.5^2 \\ &= 1.05. \end{aligned}$$

$$\begin{aligned} \mathrm{E}(X) &= np = 5 \times 0.3 = 1.5 \\ \mathrm{Var}(X) &= np(1-p) = 5 \times 0.3 \times 0.7 = 1.05 \\ \mathrm{SD}(X) &= \sqrt{1.05} = 1.024695 \end{aligned}$$

3. Let X be the number of faulty items in a sample of 10. $X \sim B(10, 0.1)$.

(a)

$$\begin{aligned} P(\text{line shut down}) &= P(X \geq 3) \\ &= 1 - P(X \leq 2) \\ &= 1 - .9298 \\ &= 0.0702 \end{aligned}$$

(b) (i) 0.3222 ; (ii) 0.6172 ; (iii) 0.8327 ; (iv) 0.9453 ; (v) 0.9877

Use `1-pbinom(2,10,p)`, p=0.1,0.2,0.3,0.4,0.5,0.6.

4.

i	0	1	2	3	4	5	Total
p_i	0.3277	0.4096	0.2048	0.0512	0.0064	0.0003	1.0000

* use check these answers with a binomial table.

$E(X) = 0 \times 0.3277 + 1 \times 0.4096 + 2 \times 0.2048 + 3 \times 0.0512 + 4 \times 0.0064 + 5 \times 0.0003 = 0.9999.$